Project No. 01-59

PROPOSED ENHANCEMENTS TO PAVEMENT ME DESIGN: IMPROVED CONSIDERATION OF THE INFLUENCE OF SUBGRADE SOILS SUSCEPTIBLE TO SHRINK/SWELL AND/OR FROST HEAVE ON PAVEMENT PERFORMANCE

APPENDIX 8

HIERARCHICAL LEVELS OF THE INPUT SOIL PARAMETERS

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TABLE OF CONTENTS

LIST OF FIGURES

List of Tables

8.0 INPUT SOIL PARAMETER HIERARCHICAL LEVELS

This document introduces the approach used to stochastica*LL*y model the variability of the required soil properties and includes the statistical input parameters required for each hierarchical level of analysis.

8.1 Summary

Randomization of the soil properties is required for the stochastic Monte Carlo volume change analyses. Random variables are generated from probability distributions. Beta distributions were generated for the required soil inputs for each hierarchal level of analysis: plasticity index (*PI*), liquid limit (*LL*), percent fines/percent passing the No. 200 sieve ($P_{\#200}$), and percent $_{clay}$/percent fine than 2 microns (P_{clay}), in situ moisture content (*w*), and dry unit weight (γ_d). The LTPP soil database (FHWA, 2010) and the NCHRP 9-23 (2006) soil databases were used to develop the statistical parameters for new subsets of soil types for the shrink/swell analysis and the frost heave analysis separately.

The U.S. Department of Agriculture (USDA) Natural Resources Conservation Service (NRCS) Soil Survey Laboratory (SSL) soil database was used to explore correlations and relationships between required input soil index properties: *PI*, *LL*, $P_{\#200}$, and P_{clay}. Due to significant correlation between the input soil index properties, an algorithmic approach was developed to randomly generate each property.

General laboratory investigations for a given project provide average values of geotechnical properties which are used as input into a deterministic solution in which only a mean value is produced. To obtain a stochastic answer, the dimensionless coefficient of variation (*CV*) is used to characterize the randomness and uncertainty in the measured properties. The coefficient of variation is generated through replicates of the test results which can require either more time and money for sampling/testing, or historical project data variance. If the coefficients of variation for the required soil properties of a project are known, sampling/testing can be reduced while increasing or maintaining the same level of confidence in the analyses and designs (provided the engineering team is experienced in statistical/stochastic analyses). As such, the coefficients of variation for the soil properties are used as key soil inputs for the stochastic volume change analyses.

8.2 Objectives

The following objectives were completed as part of this study:

- Review of the existing hierarchical levels of the descriptive statistics for the soil properties using the EICM and MEPDG.
- Evaluation of the applicability of the existing models for use in the stochastic volume change models (shrink/swell and frost heave).
- Adjustment of the datasets used for the hierarchical levels of the descriptive statistics to better represent the common soil types susceptible to shrink/swell potential and frost heave separately.
- Exploration of issues pertaining to correlation and fixed ranges of soil properties which arise during the random generation of input values required for stochastic modeling.
- Development of an algorithmic process to produce random combinations of the required soil inputs within the natural ranges and correlations.
- Validation of the algorithmic approach to produce natural combinations of randomized soil properties for input into the stochastic model.

8.3 Existing Hierarchical Soil Property Statistics used in MEPDG

The MEPDG incorporates the statistical parameters for the necessary soil parameters for each hierarchical level of analyses developed by Rosenbalm (2011) using data obtained from the LTPP DataPave library and the NCHRP 9-23A database, which was a processes version of the U.S. Department of Agriculture's (USDA) Natural Resources Conservation Service (NRCS) soil database (FHWA, 2010; Zapata, 2011).

- Rosenbalm (2011) further defines the process for generating Level 1 parameters as site specific data consisting of the average value and the coefficient of variation measured from extensive lab or in situ testing. In essence, the design engineer performing the analysis must have a high level of confidence that the statistical distributions of measured soil properties (sample) statistically represent nearly all conditions at the site (population). Rosenbalm (2011) extracted data from the LTPP SPS-1 for 28 granular soils and 28 fine grained soils from the seven states and developed pooled coefficients of variation to represent the variability in site-specific data. The existing MEPDG provides example values for coefficient of variation for Level 1 analysis for the user.

Rosenbalm (2011) breaks down the Level 3 statistics into two subgroups, referred to as Level 3A and Level 3B. For Level 3B analyses, the descriptive statistics for each AASHTO classification which are presented in **Error! Reference source not found.** through **Error! Reference source not found.**. In Level 3A, the AASTO classifications are divided into 5 groups defined as: granular base material, granular subbase/subgrade material, fine grained material, "clayey" fine grained material, and "silty" fine grained material which are presented in **Error! Reference source not found.** through **Error! Reference source not found.**, respectively. The granular base material is a grouping of the A-l-a and the A-l-b soils. The granular subbase/subgrade material is a grouping of all of A-1, A-2, and A-3 soils. The fine-grained soils included all A-4, A-5, A-6, and A-7's soils. The "clayey" fine grained group included only the A-6 and the A-7 while the "silty" fine grained group included only the A-4 and A-5 soils.

Each table provides the data count (#) mean (μ), variance (σ^2), standard deviation (σ), coefficient of variation (CV), minimum value (a), maximum value (b). Note that all parameters are in units of percent, with the exceptions of specific gravity (G_s) and *wPI* which are unitless and the particle diameter corresponding to 60% passing (D_{60}) which is in millimeters (mm). The skewness ($E[X^3]$) and kurtosis ($E[X^4]$) parameters provided by Rosenbalm (2011) were not included in the summary tables as they are not applicable to the approach used in this study. The two Beta shape factors alpha (α) and beta (β) provided by Rosenbalm (2011) were also not included in the summary table for clarity because as an updated method for estimating the shape factors was used in this study.

1.3.1 Rosenbalm (2011) Level 3a Statistical Parameters by AASHTO Classification

Table 8- 1 A-1-a Soil Properties (Rosenbalm, 2011)

	$P_{2.0"}$	$P_{1.5"}$	$P_{1.0"}$	$P_{0.5"}$	$P_{\#40}$	$P_{\#60}$	$P_{\#200}$	PI	G_s	D_{60}	*wPI*
#	1213	1213	1213	1213	2175	2175	2175	2175	489	1271	2175
μ	97.9	96.0	90.91	71.34	19.6	15.5	8.72	0.75	2.702	9.15	0.08
σ^2	43.8	80.5	144.40	188.82	38.72	26.05	14.14	2.19	0.01	42.34	0.03
σ	6.62	8.97	12.02	13.74	6.22	5.10	3.76	1.48	0.11	6.51	0.17
CV	6.8	9.3	13.2	19.3	31.7	32.9	43.1	195	4.0	71.1	206.4
a	30.0	28.0	25.0	8.0	0.0	0.0	0.0	0.0	2.243	2.4	0.000
b	100	100	100	99.0	30.0	25.9	15.0	6.0	3.152	72.2	0.894

Table 8- 2 A-1-b Soil Properties (Rosenbalm, 2011)

	$P_{2.0"}$	$P_{1.5"}$	$P_{1.0"}$	$P_{0.5"}$	$P_{\#40}$	$P_{\#60}$	$P_{\#200}$	PI	G_s	D_{60}	*wPI*
#	1033	1033	1033	1033	2610	2610	2610	2610	472	1939	2610
μ	99.1	98.1	95.1	83.99	35.50	28.51	16.52	1.492	2.661	2.824	0.308
σ^2	7.17	13.39	32.90	80.26	59.61	46.47	39.35	3.59	0.01	4.70	0.17
σ	2.68	3.66	5.74	8.96	7.72	6.82	6.27	1.89	0.10	2.17	0.41
CV	2.7	3.7	6.0	10.7	21.8	23.9	38.0	127	3.7	76.8	132
a	74.0	73.0	69.0	57.0	7.5	5.6	0.2	0.0	2.243	0.6	0.000
b	100.0	100.0	100.0	100.0	50.0	44.2	25.0	6.0	3.025	13.9	1.500

Table 8- 3 A-2-4 Soil Properties (Rosenbalm, 2011)

	$P_{2.0"}$	$P_{1.5"}$	$P_{1.0"}$	$P_{0.5"}$	$P_{\#40}$	$P_{\#60}$	$P_{\#200}$	PI	G_s	D_{60}	*wPI*
#	1683	1683	1683	1683	4218	4218	4218	4218	691	3282	4218
μ	99.5	99.1	98.0	93.9	56.08	46.03	26.7	4.24	2.677	1.34	1.16
σ^2	8.81	13.2	24.9	94.5	401	195.40	46.64	10.48	0.00	6.53	0.84
Σ	2.97	3.64	4.99	9.72	20.03	13.98	6.83	3.24	0.07	2.56	0.92
CV	3.0	3.7	5.1	10.4	35.7	30.4	25.6	76.4	2.6	191	78.9
a	54.0	44.0	36.0	31.0	8.0	6.6	2.8	0.0	2.445	0.1	0.000
B	100.0	100.0	100.0	100.0	99.0	98.3	35.4	10.0	2.975	54.9	3.500

Table 8- 4 A-2-5 Soil Properties (Rosenbalm, 2011)

	$P_{2.0"}$	$P_{1.5"}$	$P_{1.0"}$	$P_{0.5"}$	$P_{\#40}$	$P_{\#60}$	$P_{\#200}$	PI	G_s	D_{60}	*wPI*
#	561	561	561	561	1219	1219	1219	1219	5	1186	1219
μ	100	100	99.9	99.8	69.0	55.0	22.06	0.075	2.835	0.431	0.021
σ^2	0.00	0.00	0.50	3.07	189.9	85.85	39.76	0.39	0.00	0.34	0.03
σ	0.00	0.00	0.71	1.75	13.78	9.27	6.31	0.62	0.05	0.58	0.18
CV	0.0	0.0	0.7	1.8	20.0	16.8	28.6	828	1.8	135	848
a	100	100	92.0	81.0	30.0	27.8	10.5	0.0	2.781	0.2	0.000
b	100	100	100.0	100.0	97.5	74.8	35.0	10.0	2.877	4.8	3.250

Table 8- 5 A-2-6 Soil Properties (Rosenbalm, 2011)

	$P_{2.0"}$	$P_{1.5"}$	$P_{1.0"}$	$P_{0.5"}$	$P_{\#40}$	$P_{\#60}$	$P_{\#200}$	PI	G_s	D_{60}	*wPI*
#	209	209	209	209	729	729	729	729	77	330	729
μ	99.6	98.4	95.6	85.2	41.1	36.16	26.99	14.15	2.653	3.003	3.839
σ^2	2.76	11.29	36.79	173.7	256.9	139.46	44.63	6.42	0.00	12.41	1.60
σ	1.66	3.36	6.07	13.18	16.03	11.81	6.68	2.53	0.05	3.52	1.26
CV	1.7	3.4	6.3	15.5	39.0	32.7	24.8	17.9	2.0	117	32.9
a	90.0	78.0	67.0	45.0	10.0	9.3	2.8	10.5	2.507	0.1	0.448
b	100	100	100.0	100.0	99.0	78.8	35.4	25.0	2.780	19.4	8.073

Table 8- 6 A-2-7 Soil Properties (Rosenbalm, 2011)

	$P_{2.0"}$	$P_{1.5"}$	$P_{1.0"}$	$P_{0.5"}$	$P_{\#40}$	$P_{\#60}$	$P_{\#200}$	PI	G_s	D_{60}	*wPI*
#	28	28	28	28	174	174	174	174	1	46	174
μ	96.8	94.9	89.25	76.89	35.7	33.4	28.4	24.66	2.767	5.758	7.026
σ^2	22.54	43.83	124	254	52.08	40.91	31.21	47.11	N/A	23.36	6.28
σ	4.75	6.62	11.12	15.95	7.22	6.40	5.59	6.86	N/A	4.83	2.51
CV	4.9	7.0	12.5	20.7	20.2	19.2	19.6	27.8	N/A	83.9	35.7
a	86.0	82.0	70.0	50.0	15.0	13.2	8.6	12.5	2.767	0.4	1.892
b	100	100.0	100.0	100.0	60.0	51.0	35.3	50.0	2.767	18.4	16.95

Table 8- 7 A-3 Soil Properties (Rosenbalm, 2011)

	$P_{2.0"}$	$P_{1.5"}$	$P_{1.0"}$	$P_{0.5"}$	$P_{\#40}$	$P_{\#60}$	$P_{\#200}$	PI	G_s	D_{60}	*wPI*
#	508	508	508	508	689	689	689	689	92	689	689
μ	99.96	99.90	99.68	98.94	75.40	50.67	6.75	0.0	2.665	0.351	0.0
σ^2	0.31	0.61	1.70	8.66	237.25	200.87	5.41	0.0	0.00	0.03	0.0
σ	0.56	0.78	1.30	2.94	15.40	14.17	2.33	0.0	0.06	0.17	0.0
CV	0.6	0.8	1.3	3.0	20.4	28.0	34.5	0.0	0.02	49.2	0.0
a	88.0	85.0	84.0	75.0	51.0	22.0	0.3	0.0	2.445	0.1	0.0
b	100.0	100.0	100.0	100.0	100.0	96.2	10.4	0.0	2.884	2.0	0.0

Table 8- 8 A-4 Soil Properties (Rosenbalm, 2011)

	$P_{2.0"}$	$P_{1.5"}$	$P_{1.0"}$	$P_{0.5"}$	$P_{\#40}$	$P_{\#60}$	$P_{\#200}$	PI	G_s	D_{60}	*wPI*
#	1211				11002				465	11002	
μ	99.6	99.4	98.6	95.96	78.46	73.00	60.17	5.99	2.677	0.30	3.704
σ^2	2.81	3.98	8.30	32.4	215	215	295	7.96	0.00	0.51	4.85
σ	1.68	1.99	2.88	5.69	14.65	14.67	17.18	2.82	0.07	0.71	2.20
CV	1.7	2.0	2.9	5.9	18.7	20.1	28.6	47.1	0.03	239	59.5
a	86.0	83.0	79.0	64.0	36.0	36.0	35.5	0.0	2.494	0.0	0.00
b	100	100	100	100	100	99.3	99.0	10.0	2.935	10.8	9.76

Table 8- 9 A-5 Soil Properties (Rosenbalm, 2011)

	$P_{2.0"}$	$P_{1.5"}$	$P_{1.0"}$	$P_{0.5"}$	$P_{\#40}$	$P_{\#60}$	$P_{\#200}$	PI	G_s	D_{60}	*wPI*
#	31	31	31	31	332	332	332	332	14	330	332
μ	99.5	99.1	98.1	94.8	78.3	71.5	55.23	2.054	2.749	0.260	1.302
σ^2	2.92	8.69	17.66	48.47	184	179	278	8.42	0.00	0.26	3.68
σ	1.71	2.95	4.20	6.96	13.6	13.38	16.68	2.90	0.07	0.51	1.92
CV	1.7	3.0	4.3	7.3	17.3	18.7	30.2	141	0.03	195	147
a	92.0	87.0	81.0	69.0	40.0	39.3	36.3	0.0	2.620	0.0	0.000
b	100	100	100	100	100	99.3	97.5	10.0	2.869	4.8	9.250

Table 8- 10 A-6 Soil Properties (Rosenbalm, 2011)

	$P_{2.0"}$	$P_{1.5"}$	$P_{1.0"}$	$P_{0.5"}$	$P_{\#40}$	$P_{\#60}$	$P_{\#200}$	PI	G_s	D_{60}	*wPI*
#	926	926	926	926	6860	6860	6860	6860	251	6740	6860
μ	99.83	99.59	99.06	97.26	84.69	79.94	69.06	14.81	2.686	0.171	10.29
σ^2	0.89	1.94	4.81	19.0	167	178	269	8.83	0.00	0.32	11.50
σ	0.94	1.39	2.19	4.36	12.92	13.35	16.39	2.97	0.06	0.57	3.39
CV	0.9	1.4	2.2	4.5	15.3	16.7	23.7	20.1	0.02	331	33.0
a	91.0	89.0	85.0	71.0	37.5	37.5	35.6	10.5	2.507	0.0	3.885
b	100	100	100	100	100	99.4	98.2	29.0	3.089	8.8	24.36

Table 8- 11 A-7-5 Soil Properties (Rosenbalm, 2011)

	$P_{2.0"}$	$P_{1.5"}$	$P_{1.0"}$	$P_{0.5"}$	$P_{\#40}$	$P_{\#60}$	$P_{\#200}$	PI	G_s	D_{60}	*wPI*
#	58	58	58	58	683	683	683	683	17	487	683
μ	99.2	98.8	98.2	96.4	91.2	88.83	83.37	28.92	2.666	0.08	24.60
σ^2	15.41	25.29	37.47	53.15	109	122	176	83.33	0.00	0.17	95.71
σ	3.93	5.03	6.12	7.29	10.44	11.05	13.26	9.13	0.07	0.41	9.78
CV	4.0	5.1	6.2	7.6	11.5	12.4	15.9	31.6	0.03	517	39.8
a	79.0	73.0	67.0	63.0	40.0	39.3	37.0	10.5	2.605	0.0	5.52
b	100	100	100	100	100	100	100	55.0	2.875	4.8	52.25

Table 8- 12 A-7-6 Soil Properties (Rosenbalm, 2011)

	$P_{2.0"}$	$P_{1.5"}$	$P_{1.0"}$	$P_{0.5"}$	$P_{\#40}$	$P_{\#60}$	$P_{\#200}$	PI	G_s	D_{60}	*wPI*
#	618	618	618	618	4935	4935	4935	4935	141	4617	4935
μ	99.6	99.2	98.7	97.3	88.75	86.18	80.09	28.496	2.676	0.086	23.06
σ^2	3.81	7.91	14.75	31.46	138	146	193	63.06	0.00	0.20	69.70
σ	1.95	2.81	3.84	5.61	11.75	12.09	13.88	7.94	0.06	0.44	8.35
CV	2.0	2.8	3.9	5.8	13.2	14.0	17.3	27.9	0.02	516	36.2
a	83.0	75.0	70.0	66.0	40.0	39.3	36.4	14.0	2.550	0.0	6.630
b	100	100	100	100	100	99.4	99.0	75.0	2.884	8.7	66.24

1.3.2 Rosenbalm (2011) Level 3b Statistical Parameters by Generalized Material Types

Table 8- 13 Granular Base Material Soil Properties (Rosenbalm, 2011)

	$\mathbf{P}_{2.0"}$	$\mathbf{P}_{1.5"}$	$\mathbf{P}_{1.0"}$	$\mathbf{P}_{0.5"}$	$\mathbf{P}_{\#40}$	$\mathbf{P}_{\#60}$	$\mathbf{P}_{\#200}$	**PI**	$\mathbf{G}_{s}$	$\mathbf{D}_{60}$	***wPI***
#	2272				4785				961	3210	4785
μ	98.5	97.0	92.9	77.4	28.3	22.59	12.97	1.159	2.682	5.330	0.206
σ^2	27.0	50.24	96.9	182	1123	79.2	42.98	3.08	0.01	29.18	0.12
σ	5.20	7.09	9.85	13.51	10.62	8.90	6.56	1.76	0.10	5.40	0.34
CV	5.3	7.3	10.6	17.4	37.6	39.4	50.5	152	0.04	101	166
a	30.0	28.0	25.0	8.0	0.0	0.0	0.0	0.0	2.243	0.6	0.00
b	100	100	100	100	50.0	44.2	25.0	6.0	3.152	72.2	1.50

Table 8- 14 Granular Subbase and Subgrade Material Soil Properties (Rosenbalm, 2011)

	$\mathbf{P}_{2.0"}$	$\mathbf{P}_{1.5"}$	$\mathbf{P}_{1.0"}$	$\mathbf{P}_{0.5"}$	$\mathbf{P}_{\#40}$	$\mathbf{P}_{\#60}$	$\mathbf{P}_{\#200}$	**PI**	$\mathbf{G}_{s}$	$\mathbf{D}_{60}$	***wPI***
#	3062				7029				866	5533	7029
μ	99.7	99.4	98.4	95.2	58.16	46.70	24.00	4.632	2.675	1.158	1.272
σ^2	5.41	8.96	19.82	85.62	420	197	76.38	33.30	0.00	5.47	2.77
σ	2.33	2.99	4.45	9.25	20.49	14.03	8.74	5.77	0.07	2.34	1.66
CV	2.3	3.0	4.5	9.7	35.2	30.0	36.4	125	0.03	202	131
a	54.0	44.0	36.0	31.0	8.0	6.6	0.3	0.0	2.445	0.1	0.00
b	100	100	100	100	100	98.3	35.4	50.0	2.975	54.9	16.95

Table 8- 15 Fine Grained Material Soil Properties (Rosenbalm, 2011)

	$\mathbf{P}_{2.0"}$	$\mathbf{P}_{1.5"}$	$\mathbf{P}_{1.0"}$	$\mathbf{P}_{0.5"}$	$\mathbf{P}_{\#40}$	$\mathbf{P}_{\#60}$	$\mathbf{P}_{\#200}$	**PI**	$\mathbf{G}_{s}$	$\mathbf{D}_{60}$	***wPI***
#	11206						23814		888		23814
μ	99.9	99.9	99.7	99.2	82.75	78.16	67.45	13.80	2.680	0.214	10.18
σ^2	0.78	1.38	2.84	9.64	201	218	330	104	0.00	0.39	83.61
σ	0.88	1.17	1.68	3.11	14.17	14.75	18.17	10.18	0.07	0.63	9.14
CV	0.9	1.2	1.7	3.1	17.1	18.9	26.9	73.7	0.02	292	89.8
a	79.0	73.0	67.0	63.0	10.0	36.0	35.5	0.0	2.494	0.0	0.000
b	100	100	100	100	100	100	100	75.0	3.089	10.8	66.24

Table 8- 16 "Clayey" Fine Grained Material Soil Properties (Rosenbalm, 2011)

	$P_{2.0"}$	$P_{1.5"}$	$P_{1.0"}$	$P_{0.5"}$	$P_{\#40}$	$P_{\#60}$	$P_{\#200}$	PI	G_s	D_{60}	*wPI*
#	1604				12480				409	12480	
μ	99.7	99.4	98.9	97.3	86.65	82.89	74.21	21.0	2.682	0.134	16.12
σ^2	2.55	5.11	9.84	25.0	157	173	266	81.07	0.00	0.27	80.84
σ	1.60	2.26	3.14	5.00	12.54	13.17	16.32	9.00	0.06	0.52	8.99
CV	1.6	2.3	3.2	5.1	14.5	15.9	22.0	42.9	0.02	386	55.8
a	79.0	73.0	67.0	63.0	37.5	37.5	35.6	10.5	2.507	0.0	3.885
b	100	100	100	100	100	100	100	75.0	3.089	8.8	66.24

Table 8- 17 "Silty" Fine Grained Material Soil Properties (Rosenbalm, 2011)

	$P_{2.0"}$	$P_{1.5"}$	$P_{1.0"}$	$P_{0.5"}$	$P_{\#40}$	$P_{\#60}$	$P_{\#200}$	PI	G_s	D_{60}	*wPI*
#	4606				11334				479	11334	
μ	99.9	99.8	99.6	98.9	78.46	72.95	60.02	5.873	2.679	0.298	3.63
σ^2	0.79	1.17	2.68	12.11	214	214	295	8.41	0.01	0.50	4.98
σ	0.89	1.08	1.64	3.48	14.62	14.64	17.19	2.90	0.07	0.71	2.23
CV	0.9	1.1	1.6	3.5	18.6	20.1	28.6	49.4	0.03	238	61.4
a	86.0	83.0	79.0	64.0	10.0	36.0	35.5	0.0	2.494	0.0	0.00
b	100	100	100	100	100	99.3	99.0	10.0	2.935	10.8	9.76

8.4 Proposed Hierarchical Soil Parameter Statistics for Volume Change Analyses

The proposed hierarchical soil parameter statistics for shrink swell and frost heave analyses were developed by:

- First, further defining the process to obtain project related coefficients of variation for Level 1 for the volume change analyses.
- Second, adopting the descriptive statistics and classification groups defined by Rosenbalm (2011).
- Third, defining new groups of soil types which better represent the variations of shrink/swell prone soils and frost heave prone soils for Level 3.
- Fourth, generating descriptive statistics for new soil groups.

The soil parameters required as input for the shrink/swell analyses over a*LL* three hierarchical levels include the index properties of Plasticity Index (PI), Liquid Limit (*LL*), percent fines or passing the No. 200 sieve ($P_{\#200}$), percent $_{clay}$ (P_{clay}), the in situ properties of dry density (γ_d) and moisture content (w), and the engineering property used to relate soil suction changes to volumetric strain, referred to in this report as the suction compression index (SCI).

8.1.

1.4.1 Hierarchical Soil Property Statistics for Level 1 Shrink Swell and Frost Heave Analyses

Although the existing MEPDG provides example values for coefficient of variation for Level 1 analyses, such values will not be provided for the proposed shrink swell and frost heave analyses. There have been tremendous advancements in the recent decade on quantifying site-specific geotechnical uncertainty (Phoon and Kulhawy, 1999; Fenton and Griffiths 2008, Medina-Cetina and Esmailzadeh, 2014; Gong et al. 2017; Zhang et al. 2022). As such, Level 1 analyses will be reserved for projects which have sufficient site-specific information for the design engineer to produce representative distribution of the soil input parameters.

If the design engineer does not have a high level of confidence that the project data adequately represents the site-specific variability, they may choose to apply the Level 2 or Level 3 coefficients of variation to their site-specific average values. If the site-specific coefficient of variation is greater than the provided Level 2 or Level 3 values, it will be up to the design engineer's judgement on which coefficient of variation to apply. Such scenarios should warrant additional field and lab investigations as the site conditions have higher variability than the pooled LTTP data from across the US, most likely indicating a complex geology at project site with a potentia*LL*y substantial mix of coarse- and fine-grained soils.

1.4.2 In situ dry density and moisture content for Level 2 and Level 3 analyses

Unfortunately, the LTTP database lacks sufficient data to generate representative statistics for in situ dry unit weight and moisture content. This is issue is not new and was addressed in the EICM update summarized in the NCHRP 1-40D report (Witczak, 2006) by producing models that related in situ moisture and density to Proctor Compaction results and index properties using the Perera (2003) dataset which consisted of 30 sites corresponding to 143 soils. This database was used in this study to be used to represent the variance of the in situ dry density and moisture content parameters for Level 2 and Level 3 inputs. The Perera (2003) dataset was divided into the same Level 3B groups as defined by Rosenbalm (2011) and the descriptive statistics for in situ dry density and the moisture content were calculated, which are presented in **Error! Reference source not found.**.

Table 8- 18 Descriptive Statistics for In Situ Dry Unit Weight and Moisture Content

	A-1		**A-1, A-2, & A-3**		**A-4, A-5, A-6 & A-7**		**A-4 & A-5**		**A-6 & A-7**	
	w	**γ_d**	**w**	**γ_d**	**w**	**γ_d**	**w**	**γ_d**	**w**	**γ_d**
#	14		23		120		34		86	
μ	14	14	7.99	119.18	19.53	106.03	17.57	106.11	20.30	106.00
σ^2	6.62	127.60	7.88	604.59	27.47	83.82	32.11	95.80	23.86	80.15
σ	1.22	75.59	2.81	24.59	5.24	9.16	5.67	9.79	4.88	8.95
CV	1.10	8.69	35.13	20.63	26.84	8.63	32.24	9.22	24.06	8.45
a	16.68	6.81	5.33	14.52	8.84	82.24	8.84	83.00	10.13	82.24
b	5.33	108.46	14.87	141.14	35.21	126.11	31.76	124.36	35.21	126.11
α	22.26	88.66	5.56	3.25	7.85	60.85	5.57	51.30	9.86	63.72
β	37.79	62.73	14.38	0.68	11.51	51.37	9.05	40.51	14.45	53.95

For Level 3 analyses, if the average (input) value of the in situ moisture content of the subgrade is unknown the user must assume that it is equivalent to the optimum moisture content, which is genera*LL*y true for new construction. If the user is running an exploratory or preliminary analysis and the optimum moisture content of the subgrade is also unknown, the program will estimate an optimum moisture content using index property and correlation models from the EICM.

Compaction Model for Granular Materials (wPI = 0)

The maximum dry unit weight for compacted materials is expressed as:

$$\gamma_{\text{d max comp mod}} = \frac{G_s \gamma_{water}}{1 + \frac{w_{opt} G_s}{S_{opt}}} \tag{8-1}$$

Where: $\gamma_{d\ max\ comp\ mod}$ = Maximum dry density by Modified proctor (pcf), G_s = Specific gravity, γ_{water} = Unit weight of water (pcf), w_{opt} = Optimum gravimetric moisture content by Modified proctor (%), and S_{opt} = Degree of saturation at optimum conditions (%).

The relationship between optimum gravimetric moisture content and gradation data for granular materials is expressed as:

$$w_{opt} = -120.14 - 0.06766 P_{1.5"} + 3.7269 D_{60} - 0.167 P_{40} + 0.117 P_{60} + 142.53 e^{(-0.0389 * D_{60})} \tag{8-2}$$

Where: $P_{1.5"}$ = Percent passing 1.5" (%), P_{40} = Percent passing #40 US sieve (%), P_{60} = Percent passing #60 US sieve (%), and D_{60} = Diameter corresponding to 60% passing material (mm).

The relationship for saturation at optimum conditions given gradation data for granular materials is expressed as:

$$S_{opt} = -100.17 + 1.4991 P_{2"} + 0.56155 P_{1"} - 0.36755 P_{0.5"} \tag{8-3}$$

Where: $P_{2"}$ = Percent passing 2" (%), $P_{1"}$ = Percent passing 1" (%), and $P_{0.5"}$ = Percent passing 0.5" (%).

For compacted materials, the dry unit weight is assumed to be equal to the maximum dry unit weight found above.

$$\gamma_d = \gamma_{d\ maxcomp\ mod} \tag{8-4}$$

Compaction Model for Plastic Materials (wPI > 0)

The relationship between the gravimetric optimum water content and soil index properties is expressed as:

$$w_{opt} = 8.3932 wPI_{adj}^{0.3075} \tag{8-5}$$

Where: wPI_{adj} = an adjusted PI value applicable only to the empirical correlations for the optimum water content and the maximum dry unit weight for plastic soils presented in this section.

The adjusted *wPI* is expressed as:

$$wPI_{adj} = \frac{PI_{adj} P_{200}}{100} \tag{8-6}$$

Where: PI_{adj} = an adjusted PI value. If wPI_{adj} <1 then wPI_{adj} = 1

The adjusted PI value is expressed as:

$$PI_{adj} = e^{\frac{P_{200}+42.13}{33.94}} \tag{8-7}$$

If $PI > PI_{adj}$, then use PI_{adj} and if $PI \leq PI_{adj}$, then $PI_{adj} = PI$.

For wPI_{adj} values that are equal to 1, both the predicted optimum water content for low-plasticity materials (equation above) and the water content predicted for non-plastic materials should be calculated and an average value used. The maximum dry density for fine grained materials can be expressed in terms of the optimum moisture content as:

$$\gamma_{d\max\_comp\_std} = 142.115 - 1.959 w_{opt} \tag{8-8}$$

Where: $\gamma_{d\ max\ comp\ std}$ = maximum dry density by Standard proctor (pcf).

For uncompacted materials, the dry unit weight was related to the maximum dry unit weight from the the Standard proctor:

$$\gamma_d = 1.0156 \gamma_{d\ maxcomp\ std} - 2.464 \tag{8-9}$$

1.4.3 Hierarchical Soil Property Statistics for Frost Heave Analysis

The classification groups defined by Rosenbalm (2011) were adopted for the frost heave analysis as most soils are susceptible to frost action with the exception of gravels and clean sands. The five Level 3B generalized groups of soil types defined by Rosenbalm (2011) were chosen to be used for the Level 3 analysis of frost heave as they have the largest variation. The Level 3A groups by AASTO classification were chosen to be used for the Level 2 analysis of frost heave. The dry unit weight and the in-situ moisture content statistical parameters determined from the Perera (2003) soil database with the Level 3B groups (**Error! Reference source not found.**) as defined by Rosenbalm (2011) were used for both the Level 2 and Level 3 analyses. The alpha and beta shape factors were calculated using the method presented in the literature review (Appendix 1). To eliminate unnatural "U-shaped" distribution, the beta shape factor was corrected to 1 if both shape factors were initia*LL*y less than 1, and the alpha shape factor was less than the beta shape factor.

Level 3 Statistical Parameters for SS Analysis

Note that user input is required for the expected value and coefficient of variation (or standard deviation) for the Level 1 analyses based on the site-specific data.

Table 8- 19 Level 2 A-1-a Soil Properties

	PI	$P_{\#200}$	*w*	γ_d
#	2175		23	
μ	0.75	8.72	7.99	123.33
σ^2	2.19	14.14	7.88	92.54
σ	1.48	3.76	2.81	9.62
CV	197.33	43.12	35.13	7.80
a	0	0	5.33	108.46
b	6	15	14.87	141.14
α	0.10	1.67	5.56	89.12
β	1*	1.20	14.38	106.74

*Corrected to 1 to eliminate unnatural "U-shaped" distribution

Table 8- 20 Level 2 A-1-b Soil Properties

	PI	$P_{\#200}$	*w*	γ_d
#	2610		23	
μ	1.492	16.52	7.99	123.33
σ^2	3.57	39.31	7.88	92.54
σ	1.89	6.27	2.81	9.62
CV	126.68	37.95	35.13	7.80
a	0	0.2	5.33	108.46
b	6	25	14.87	141.14
α	0.22	1.72	5.56	89.12
β	1*	0.89	14.38	106.74

*Corrected to 1 to eliminate unnatural "U-shaped" distribution

Table 8- 21 Level 2 A-2-4 Soil Properties

	PI	$P_{\#200}$	*w*	γ_d
#	4218		23	
μ	4.24	26.7	7.99	123.33
σ^2	10.50	46.65	7.88	92.54
Σ	3.24	6.83	2.81	9.62
CV	76.42	25.58	35.13	7.80
a	0	2.8	5.33	108.46
B	10	35.4	14.87	141.14
α	0.56	3.35	5.56	89.12
β	1*	1.22	14.38	106.74

*Corrected to 1 to eliminate unnatural "U-shaped" distribution

Table 8- 22 Level 2 A-2-5 Soil Properties

	PI	$P_{\#200}$	*w*	γ_d
#	1219		23	
μ	0.08	22.06	7.99	123.33
σ^2	0.38	39.82	7.88	92.54
σ	0.62	6.31	2.81	9.62
CV	828.00	28.60	35.13	7.80
a	0.00	10.50	5.33	108.46
b	10.00	35.00	14.87	141.14
α	0.01	5.99	5.56	89.12
β	1.00	6.70	14.38	106.74

Table 8- 23 Level 2 A-2-6 Soil Properties

	PI	$P_{\#200}$	*w*	γ_d
#	729		23	
μ	14.15	26.99	7.99	123.33
σ^2	6.40	44.62	7.88	92.54
σ	2.53	6.68	2.81	9.62
CV	17.90	24.80	35.13	7.80
a	10.50	2.80	5.33	108.46
b	25.00	35.40	14.87	141.14
α	23.10	3.45	5.56	89.12
β	68.67	1.20	14.38	106.74

Table 8- 24 Level 2 A-2-7 Soil Properties

	PI	$P_{\#200}$	*w*	γ_d
#	174		23	
μ	24.66	28.40	7.99	123.33
σ^2	47.06	31.25	7.88	92.54
σ	6.86	5.59	2.81	9.62
CV	27.80	19.60	35.13	7.80
a	12.50	8.60	5.33	108.46
b	50.00	35.30	14.87	141.14
α	8.42	5.99	5.56	89.12
β	17.54	2.09	14.38	106.74

Table 8- 25 Level 2 A-3 Soil Properties

	PI	$P_{\#200}$	*w*	γ_d
#	689		23	
μ	0.00	6.75	7.99	123.33
σ^2	0.00	5.43	7.88	92.54
σ	0.00	2.33	2.81	9.62
CV	0.00	34.50	35.13	7.80
a	0.00	0.30	5.33	108.46
b	0.00	10.40	14.87	141.14
α	0.00	2.40	5.56	89.12
β	0.00	1.36	14.38	106.74

Table 8- 26 Level 2 A-4 Soil Properties

	PI	$P_{\#200}$	*w*	γ_d
#	11002		34	
μ	5.99	60.17	17.57	106.11
σ^2	7.95	295.15	32.11	95.80
σ	2.82	17.18	5.67	9.79
CV	47.08	28.55	17.86	7.03
a	0.00	35.50	8.84	83.00
b	10.00	99.00	31.76	124.36
α	1.21	7.11	19.02	88.72
β	0.81	11.19	30.90	70.06

Table 8- 27 Level 2 A-5 Soil Properties

	PI	$P_{\#200}$	*w*	γ_d
#	332		34	
μ	2.05	55.23	17.57	106.11
σ^2	8.41	278.22	32.11	95.80
σ	2.90	16.68	5.67	9.79
CV	141.00	30.20	17.86	7.03
a	0.00	36.30	8.84	83.00
b	10.00	97.50	31.76	124.36
α	0.19	7.26	19.02	88.72
β	1*	16.22	30.90	70.06

*Corrected to 1 to eliminate unnatural "U-shaped" distribution

Table 8- 28 Level 2 A-6 Soil Properties

	PI	$P_{\#200}$	*w*	γ_d
#	6860		86	
μ	14.81	69.06	20.30	106.00
σ^2	8.82	268.63	23.86	80.15
σ	2.97	16.39	4.88	8.95
CV	20.05	23.73	24.06	8.45
a	10.50	35.60	10.13	82.24
b	29.00	98.20	35.21	126.11
α	18.84	7.73	9.86	63.72
β	62.03	6.73	14.45	53.95

Table 8- 29 Level 2 A-7-5 Soil Properties

	PI	$P_{\#200}$	*w*	γ_d
#	683		86	
μ	28.92	83.37	20.30	106.00
σ^2	83.36	175.83	23.86	80.15
σ	9.13	13.26	4.88	8.95
CV	31.57	15.91	24.06	8.45
a	10.50	37.00	10.13	82.24
b	55.00	100.00	35.21	126.11
α	5.47	9.70	9.86	63.72
β	7.74	3.48	14.45	53.95

Table 8- 30 Level 2 A-7-6 Soil Properties

	PI	$P_{\#200}$	*w*	γ_d
#	4935		86	
μ	28.50	80.09	20.30	106.00
σ^2	63.04	192.65	23.86	80.15
σ	7.94	13.88	4.88	8.95
CV	27.86	17.33	24.06	8.45
a	14.00	36.40	10.13	82.24
b	75.00	99.00	35.21	126.11
α	9.58	9.36	9.86	63.72
β	30.74	4.05	14.45	53.95

Level 3 Statistical Parameters for FH Analysis

Table 8- 31 Level 3 Granular Base Material Properties (A-1-a & A-1-b)

	PI	$P_{\#200}$	*w*	γ_d
#	4785		14	
μ	1.159	12.97	6.62	127.60
σ^2	3.10	43.03	1.22	75.59
σ	1.76	6.56	1.10	8.69
CV	151.86	50.58	16.68	6.81
a	0	0	5.33	108.46
b	6	25	8.81	141.14
α	0.16	1.36	22.26	88.66
β	1 *	1.26	37.79	62.73

*Corrected to 1 to eliminate unnatural "U-shaped" distribution

Table 8- 32 Level 3 Granular Subbase and Subgrade Material Properties (A-1, A-2 & A-3)

	PI	$P_{\#200}$	*w*	γ_d
#	7029		23	
μ	4.632	24.00	7.99	123.33
σ^2	33.29	76.39	7.88	92.56
σ	5.77	8.74	2.81	9.62
CV	124.57	36.42	35.13	7.80
a	0	0.3	5.33	108.46
b	50	35.4	14.87	141.14
α	0.49	1.77	5.56	89.10
β	4.82	0.85	14.38	106.74

Table 8- 33 Level 3 Fine Grained Material Properties (A-4, A-5, A-6, &, A-7)

	PI	$P_{\#200}$	*w*	γ_d
#	23814		120	
μ	13.8	67.45	19.53	106.03
σ^2	103.63	330.15	27.47	83.82
σ	10.18	18.17	5.24	9.16
CV	73.77	26.94	26.84	8.63
a	0	35.5	8.84	82.24
b	75	100	35.21	126.11
α	1.32	6.46	7.85	60.85
β	5.83	6.58	11.51	51.37

Table 8- 34 Level 3 "Silty" Fine Grained Material Properties (A-4 & A-5)

	PI	$P_{\#200}$	*w*	γ_d
#	11334		34	
μ	5.873	60.02	17.57	106.11
σ^2	8.41	295.50	32.11	95.80
σ	2.9	17.19	5.67	9.79
CV	49.38	28.64	32.24	9.22
a	0	35.5	8.84	83.00
b	10	99	31.76	124.36
α	1.11	7.10	5.57	51.30
β	0.78	11.28	9.05	40.51

Table 8- 35 Level 3 "Clayey" Fine Grained Material Properties (A-4 & A-5)

	PI	$P_{\#200}$	*w*	γ_d
#	12480		86	
μ	21	74.21	20.30	106.00
σ^2	81.00	266.34	23.86	80.15
σ	9	16.32	4.88	8.95
CV	42.86	21.99	24.06	8.45
a	10.5	35.6	10.13	82.24
b	75	100	35.21	126.11
α	4.40	7.68	9.86	63.72
β	22.60	5.13	14.45	53.95

1.4.4 Hierarchical Soil Property Statistics for Level 2 and Level 3 Shrink Swell Analysis

A new set of groups of data were used to define the descriptive statistics for the Level 2 and Level 3 shrink swell analyses. Soils with a propensity to exhibit shrink swell behavior, defined by A-6 and A-7 soils with a *wPI* > 10, were extracted from the LTTP database and used as the dataset to represent the Level 3 analyses. This group of soils is similar to the "$_{clay}$ey" fine grained soil group defined by Rosenbalm (2011), but does not include A-6 and A-7 soils that may have a low PI and $P_{\#200}$; for example, a lean $_{clay}$ (CL) which is classified as an A-6 with 40% fines and a PI of 12 (i.e. *wPI* = 4.8) typica*LL*y exhibits very low expansion potential. The grouping of material for the Level 2 analysis followed a similar idea of the Level 3A groups defined by Rosenbalm (2011) but instead of grouping by AASHTO classification, the soils were grouped by *wPI* in intervals of 10. Although this may result in a very similar arrangement of the data, the *wPI* criteria will tend to group soils together that have similar expansion potentials, as *wPI* is an adequate correlation for expansion potential.

The dry unit weight and the in-situ moisture content statistical parameters had to be determined using the Perera (2003) soil database. Due to the lack of data points with high expansion potential, the data was with *wPI* > 10 was used for a*LL* Level 2 subgroups regardless of the *wPI* limits.

The soil parameters required as input for the shrink swell analysis for Level 2 and Level 3 include the index properties of percent fines or passing the No. 200 sieve ($P_{\#200}$), percent $_{clay}$ (P_{clay}), the in-situ properties of dry density (γ_d) and moisture content (w). The mean, variance, standard deviation, coefficient of variation, minimum, maximum, and the Beta distribution shape factors for the required index properties were calculated for Level 2 and Level 3 (Rosenbalm and Zapata, 2012). The statistical parameters for the maximum dry density and the optimum moisture content were referenced from the Perera (2003). The statistical parameters for Level 2 shrink swell analyses are subdivided by *wPI* and are present in **Error! Reference source not found.** through **Error! Reference source not found.**. For the statistical parameters for Level 3 shrink swell analyses, the alpha and beta shape factors were calculated using the method presented in the literature review (Appendix 1). To eliminate unnatural "U-shaped" distribution, the beta shape factor was corrected to 1 if both shape factors were initia*LL*y less than 1, and the alpha shape factor was less than the beta shape factor.

Level 3 Statistical Parameters for SS Analysis

Note that user input is required for the expected value and coefficient of variation (or standard deviation) for the Level 1 analyses based on the site-specific data.

Level 2 Statistical Parameters for SS Analysis

Table 8- 36 Level 2 Shrink/Swell Soil with 10 < wPI < 20

	LL	*PI*	$P_{\#200}$	P_{clay}	w	γ_d
#					55	
μ	46.49	23.48	66.31	30.92	22.18	103.09
σ^2	25.60	17.81	184.14	73.79	15.68	52.56
σ	5.06	4.22	13.57	8.59	3.96	7.25
CV	10.89	17.96	20.47	27.77	17.86	7.03
a	41.00	14.00	36.40	10.00	14.19	82.24
b	64.00	35.00	98.00	52.60	35.21	121.39
α	63.96	16.56	11.79	6.11	19.05	94.05
β	203.99	20.12	12.49	6.33	31.07	82.55

Table 8- 37 Level 2 Shrink/Swell Soil with with 20 ≤ wPI < 30

	LL	*PI*	$P_{\#200}$	P_{clay}	*w*	γ_d
#					55	
μ	52.17	29.61	76.98	36.58	22.18	103.09
σ^2	105.68	102.21	281.57	152.52	15.68	52.56
σ	10.28	10.11	16.78	12.35	3.96	7.25
CV	19.71	34.13	21.80	33.76	17.86	7.03
a	25.00	11.00	30.20	5.70	14.19	82.24
b	102.00	75.00	99.00	75.50	35.21	121.39
α	16.31	5.80	6.05	4.45	19.05	94.05
β	29.90	14.14	2.85	5.61	31.07	82.55

Table 8- 38 Level 2 Shrink/Swell Soil with with 30 ≤ wPI < 40

	LL	*PI*	$P_{\#200}$	P_{clay}	*w*	γ_d
#					55	
μ	61.13	39.11	88.74	46.41	22.18	103.09
σ^2	67.24	37.33	91.39	114.28	15.68	52.56
σ	8.20	6.11	9.56	10.69	3.96	7.25
CV	13.41	15.61	10.77	23.03	17.86	7.03
a	50.00	31.00	60.60	15.50	14.19	82.24
b	85.00	61.00	99.00	75.50	35.21	121.39
α	37.61	29.67	22.30	8.63	19.05	94.05
β	80.65	80.09	8.13	8.12	31.07	82.55

Table 8- 39 Level 2 Shrink/Swell Soil with 40 ≤ wPI < 50

	LL	*PI*	$P_{\#200}$	P_{clay}	*w*	γ_d
#					55	
μ	70.50	47.36	92.86	58.98	22.18	103.09
σ^2	69.89	27.46	29.05	87.42	15.68	52.56
σ	8.36	5.24	5.39	9.35	3.96	7.25
CV	11.85	11.07	5.80	15.85	17.86	7.03
a	57.00	41.00	79.40	40.70	14.19	82.24
b	83.00	61.00	98.90	68.80	35.21	121.39
α	33.72	55.34	91.39	13.26	19.05	94.05
β	31.22	118.67	41.01	7.12	31.07	82.55

Table 8- 40 Level 2 Shrink/Swell Soil with wPI ≥ 50

	LL	*PI*	$P_{\#200}$	P_{clay}	*w*	γ_d
#					55	
μ	87.20	65.40	89.48	57.52	22.18	103.09
σ^2	89.68	50.84	16.89	31.92	15.68	52.56
σ	9.47	7.13	4.11	5.65	3.96	7.25
CV	10.86	10.90	4.59	9.81	17.86	7.03
a	77.00	59.00	82.40	51.00	14.19	82.24
b	102.00	75.00	92.60	64.00	35.21	121.39
α	49.79	50.10	144.49	51.29	19.05	94.05
β	72.24	75.15	63.68	50.98	31.07	82.55

Level 3 Statistical Parameters for SS Analysis

Table 8- 41 Level 3 Shrink/Swell Soil (A-6 & A-7 with wPI > 10)

	LL	*PI*	$P_{\#200}$	P_{clay}	*w*	γ_d
#					55	
μ	45.68	25.53	77.16	33.50	22.18	103.09
σ^2	151.04	105.27	239.94	124.10	15.69	52.53
σ	12.29	10.26	15.49	11.14	3.96	7.25
CV	26.92	40.17	20.07	33.25	17.86	7.03
a	25.00	11.00	30.20	0.00	14.19	82.24
b	102.00	75.00	99.00	75.50	35.21	121.39
α	9.82	4.56	7.20	4.59	19.05	94.05
β	26.76	15.54	3.35	5.75	31.07	82.55

To provide an example of the beta distribution, which will be incorporated in the stochastic analysis, the following figures present the probability distribution plots for the Level 2 and Level 3 input parameters: *PI*, *LL*, $P_{\#200}$, P_{clay}, *w*, and γ_d.

FH Level 2 Beta Distribution for Input Soil Properties

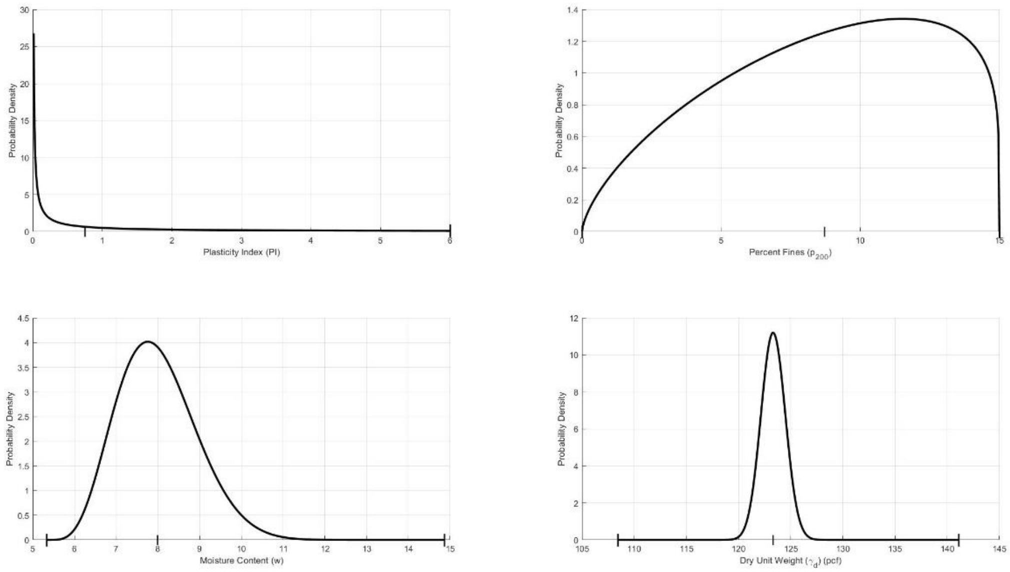


Figure 8- 1 Beta Probability Distributions for Level 2 FH with AASHTO classification “A-1-a”

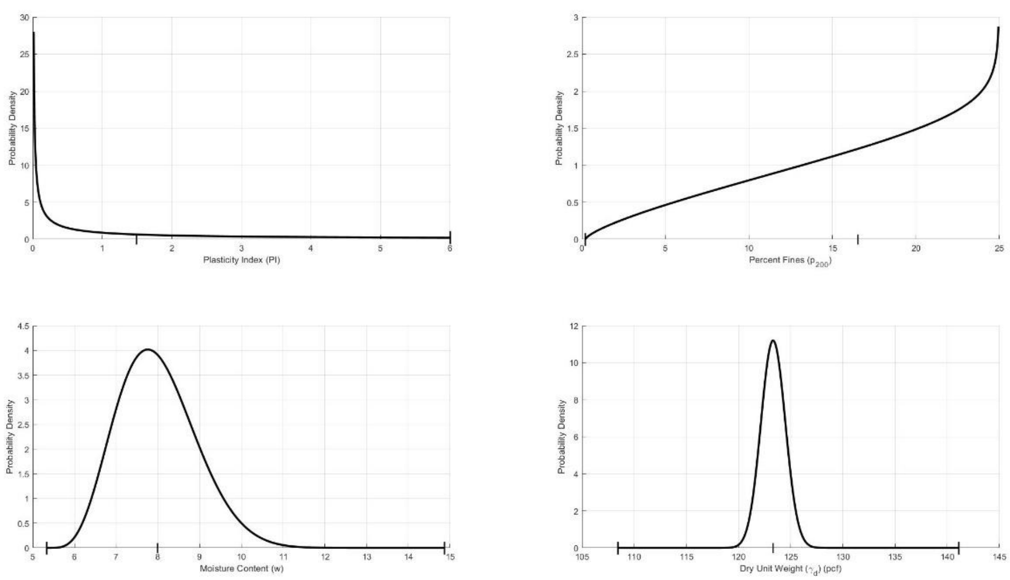


Figure 8- 2 Beta Probability Distributions for Level 2 FH with AASHTO classification “A-1-b”

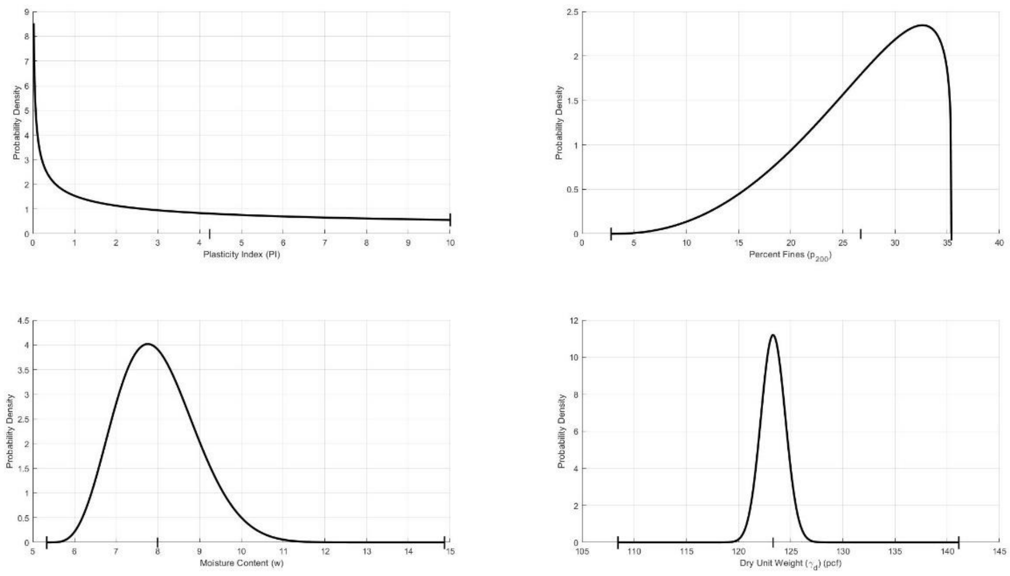


Figure 8- 3 Beta Probability Distributions for Level 2 FH with AASHTO classification "A-2-4"

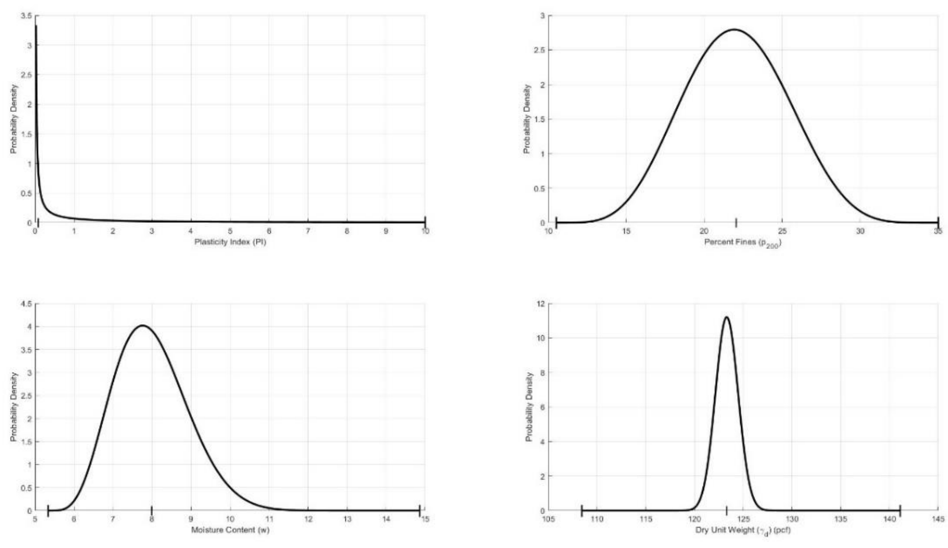


Figure 8- 4 Beta Probability Distributions for Level 2 FH with AASHTO classification "A-2-5"

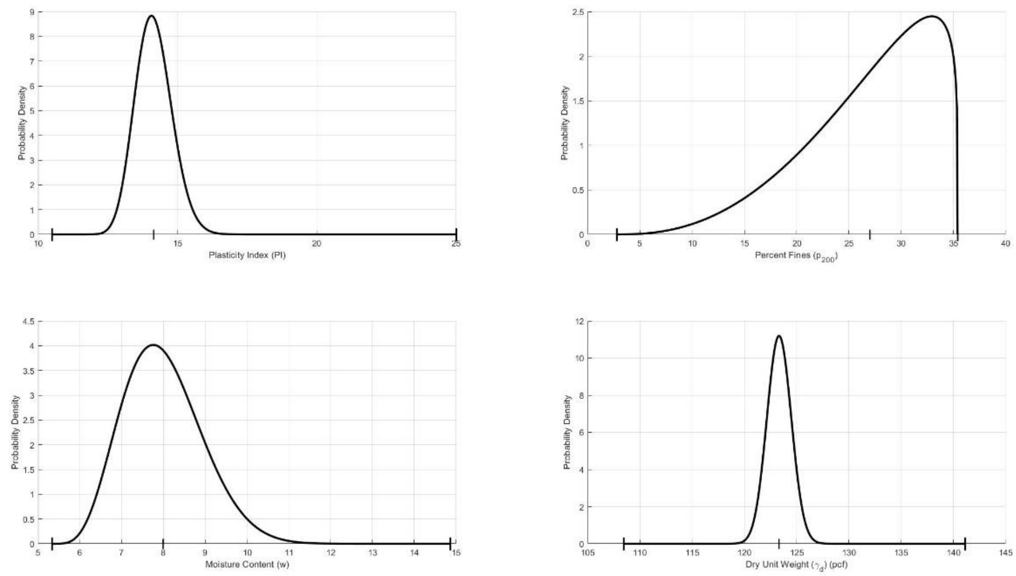


Figure 8- 5 Beta Probability Distributions for Level 2 FH with AASHTO classification “A-2-6”

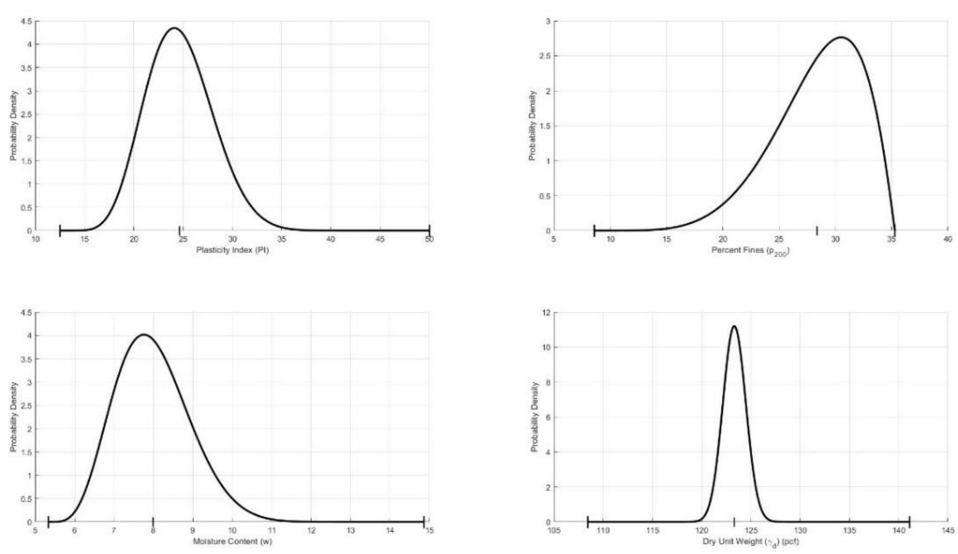


Figure 8- 6 Beta Probability Distributions for Level 2 FH with AASHTO classification “A-2-7”

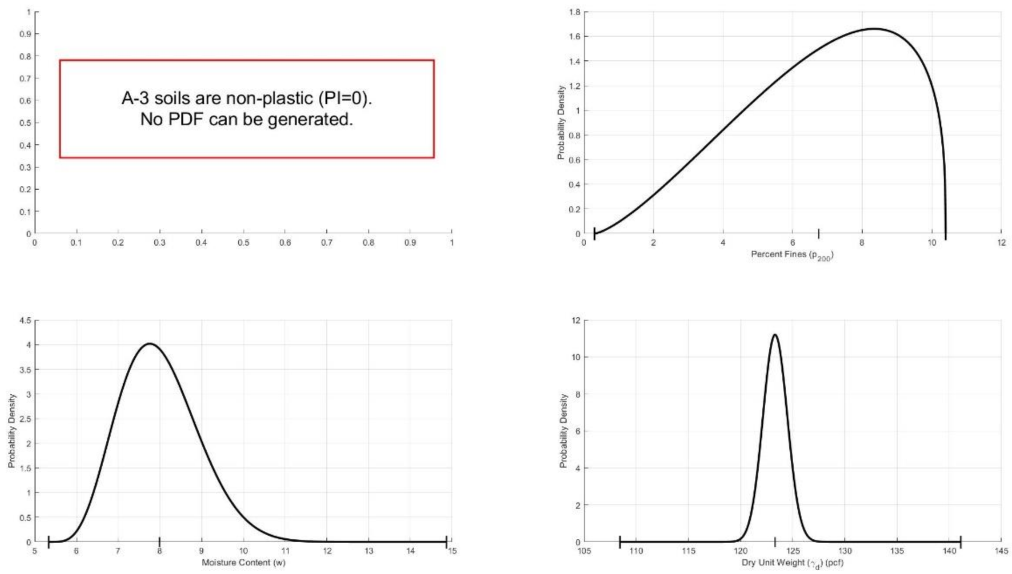


Figure 8- 7 Beta Probability Distributions for Level 2 FH with AASHTO classification "A-3"

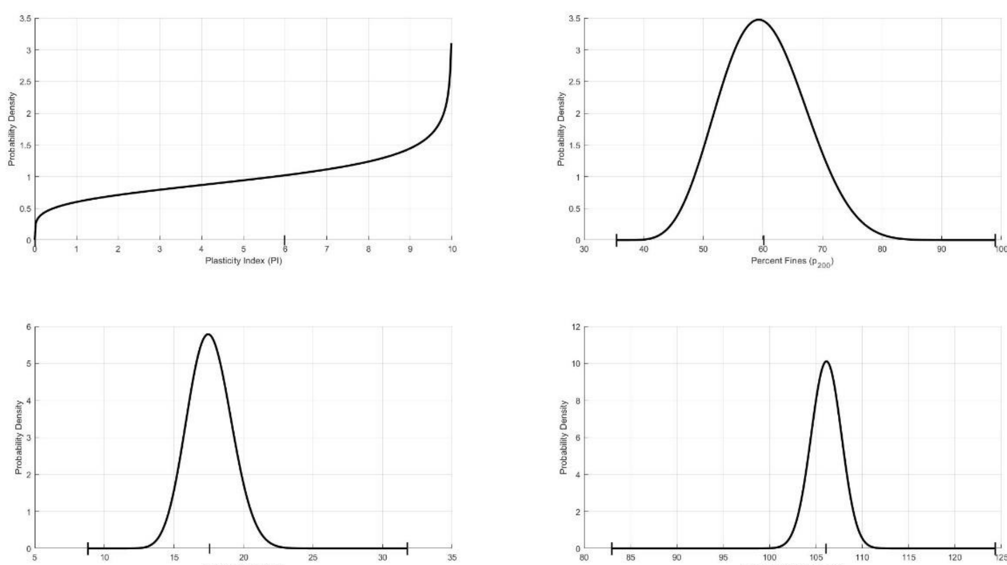


Figure 8- 8 Beta Probability Distributions for Level 2 FH with "A-4"

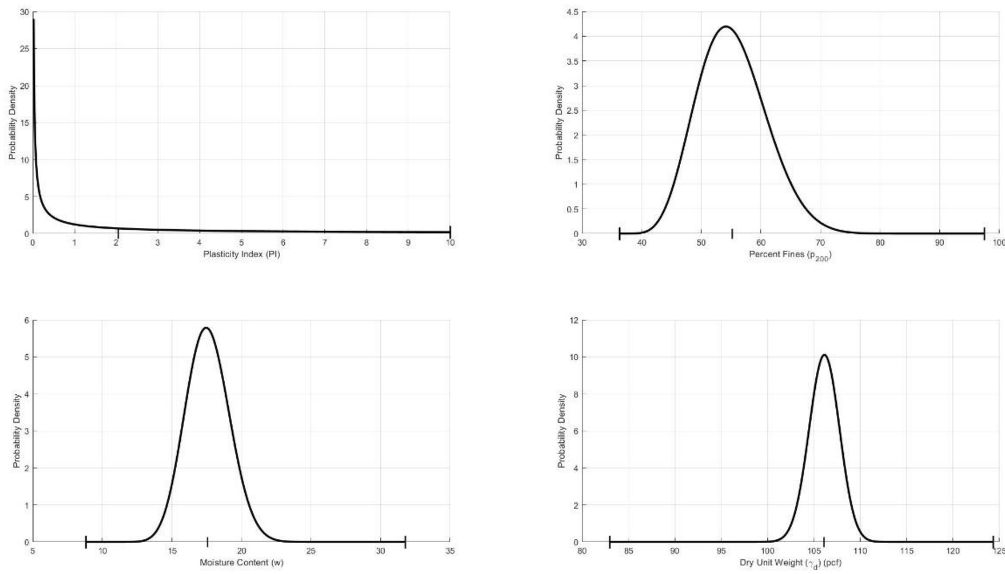


Figure 8- 9 Beta Probability Distributions for Level 2 FH with "A-5"

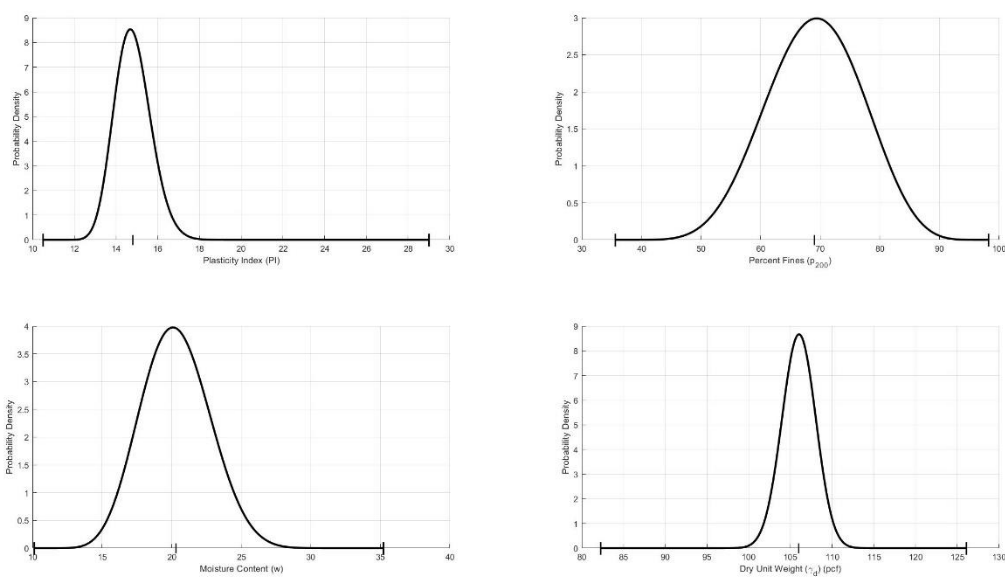


Figure 8- 10 Beta Probability Distributions for Level 2 FH with "A-6"

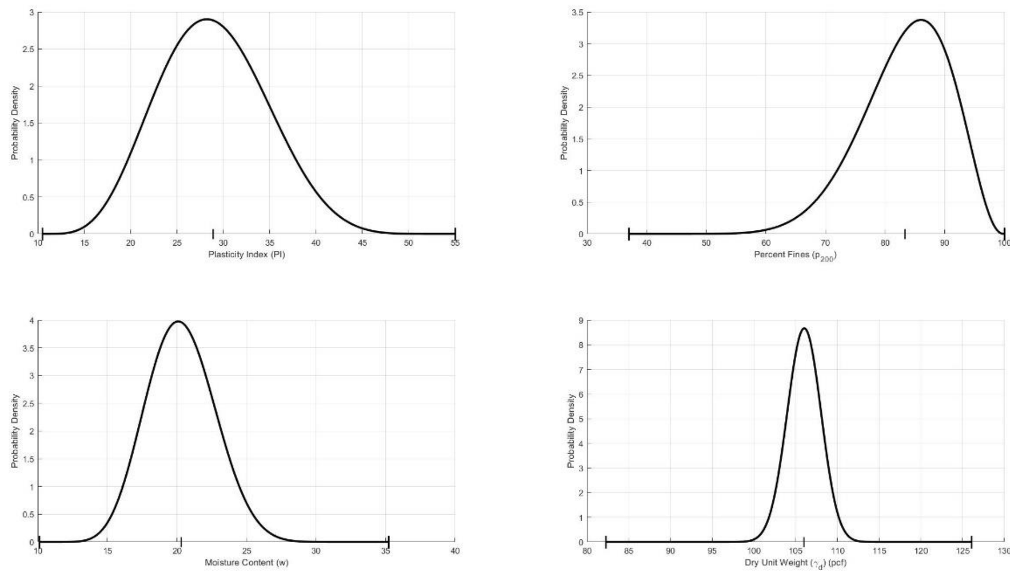


Figure 8- 11 Beta Probability Distributions for Level 2 FH with "A-7-5"

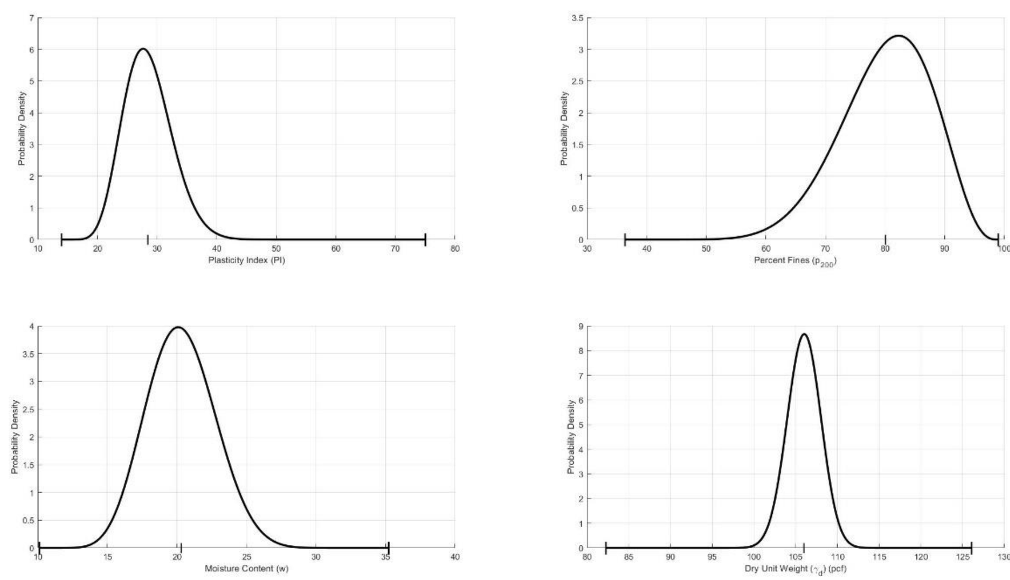


Figure 8- 12 Beta Probability Distributions for Level 2 FH with "A-7-6"

SS Level 3 Beta Distribution for Input Soil Properties

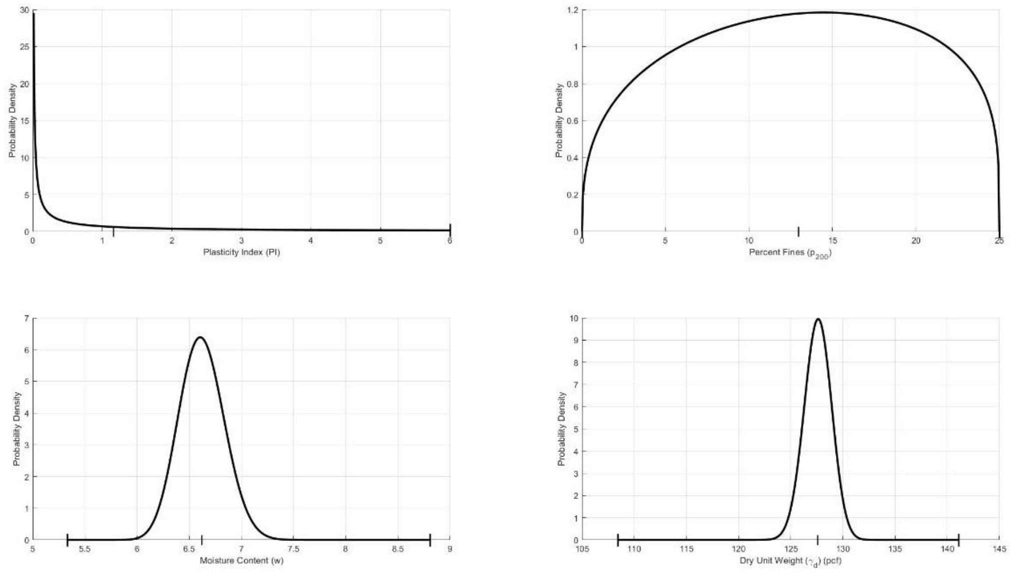


Figure 8- 13 Beta Probability Distributions for Level 3 FH with "A-1-a & A-1-b"

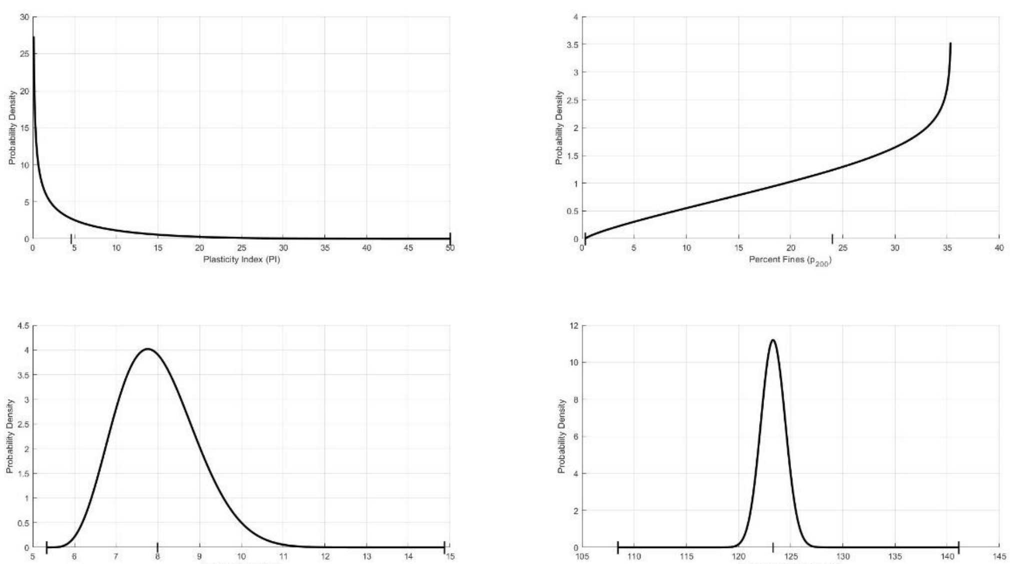


Figure 8- 14 Beta Probability Distributions for Level 3 FH with "A-1, A-2, & A-3"

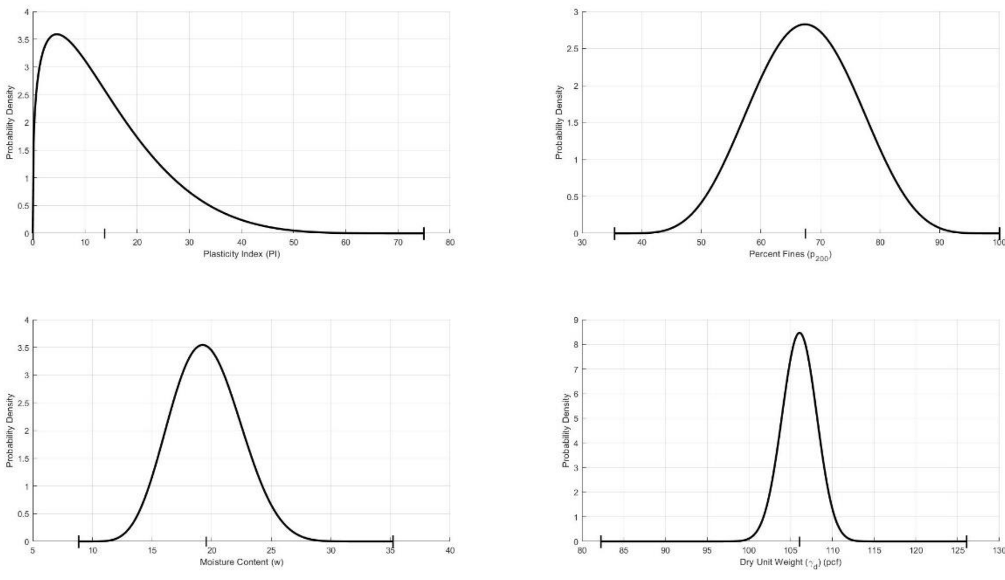


Figure 8- 15 Beta Probability Distributions for Level 3 FH with "A-4, A-5, A-6 & A-7"

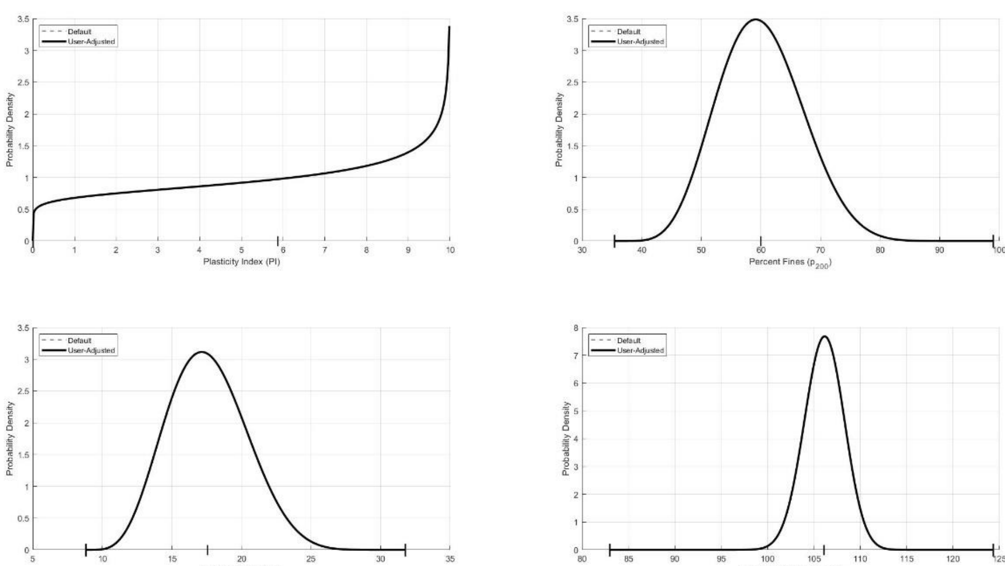


Figure 8- 16 Beta Probability Distributions for Level 3 FH with "A-4 & A-5"

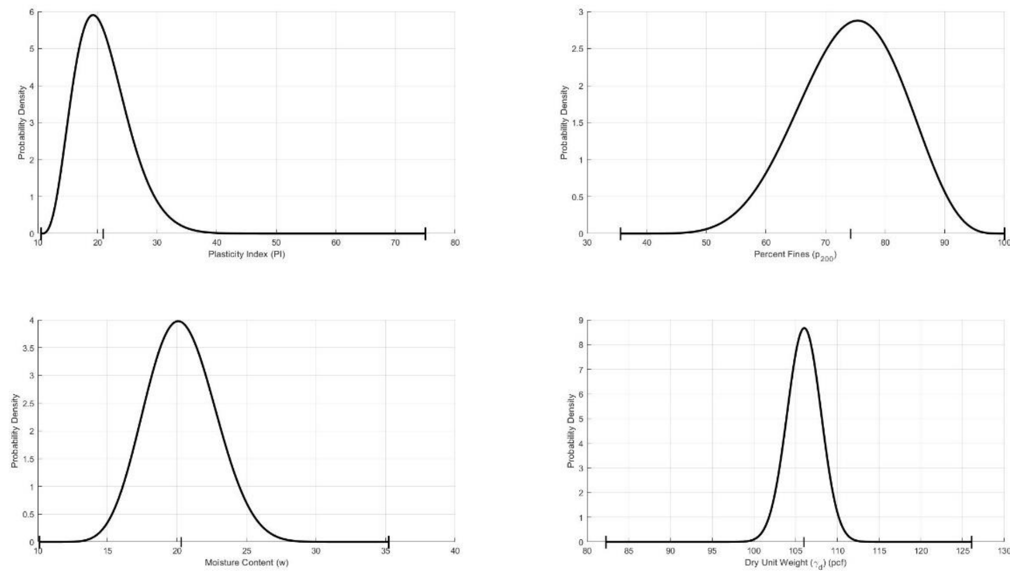


Figure 8- 17 Beta Probability Distributions for Level 3 FH with "A-6 & A-7"

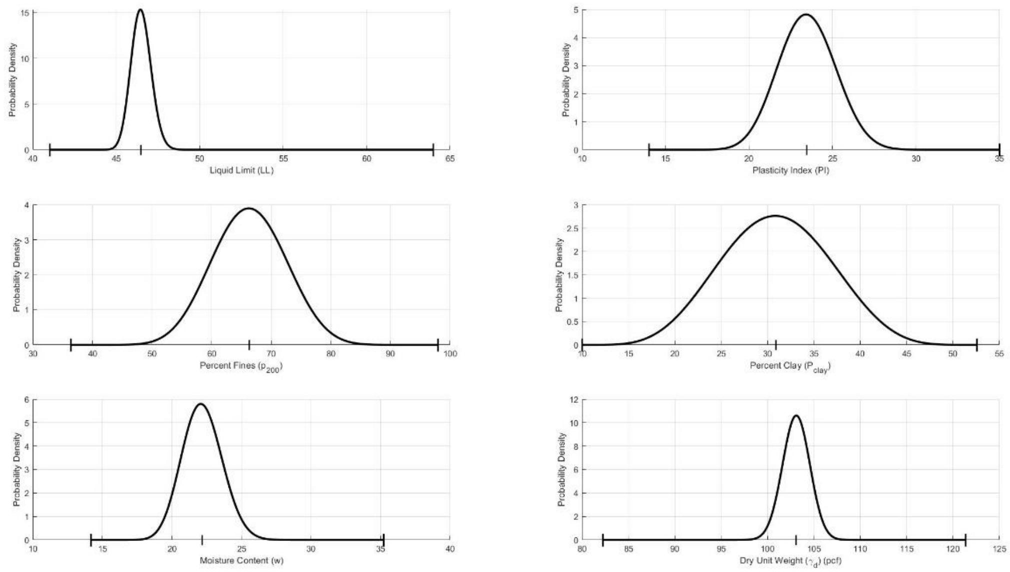


Figure 8- 18 Beta Probability Distributions for Level 2 SS Input with 10 ≤ wPI < 20

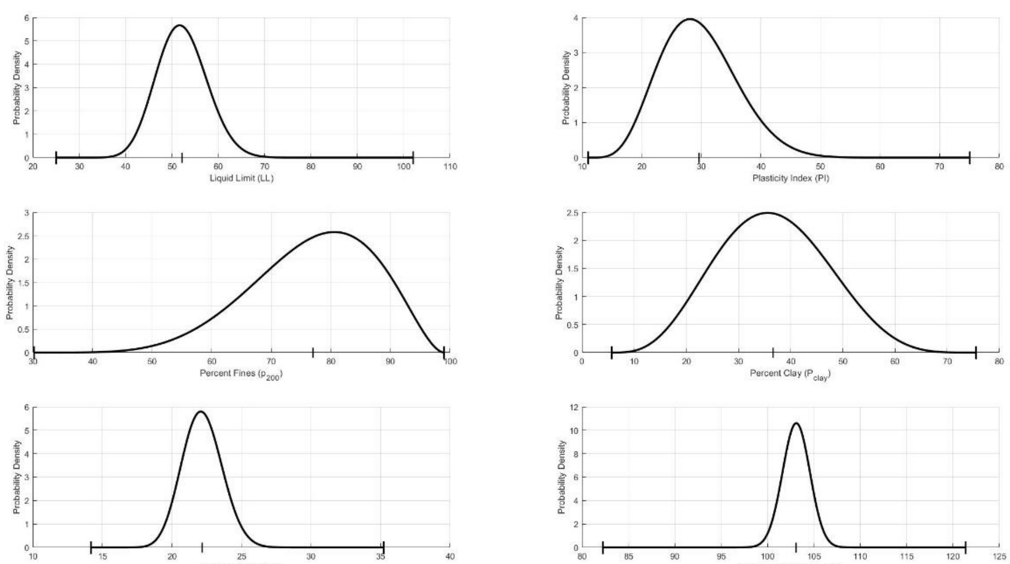


Figure 8- 19 Beta Probability Distributions for Level 2 SS with 20 ≤ wPI < 30

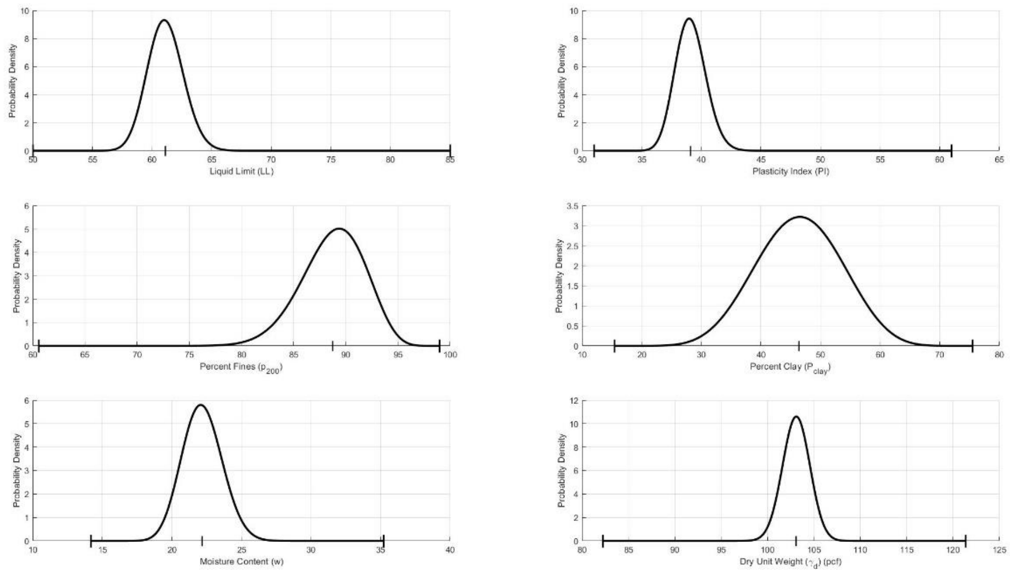


Figure 8- 20 Beta Probability Distributions for Level 2 SS with 30 ≤ wPI < 40

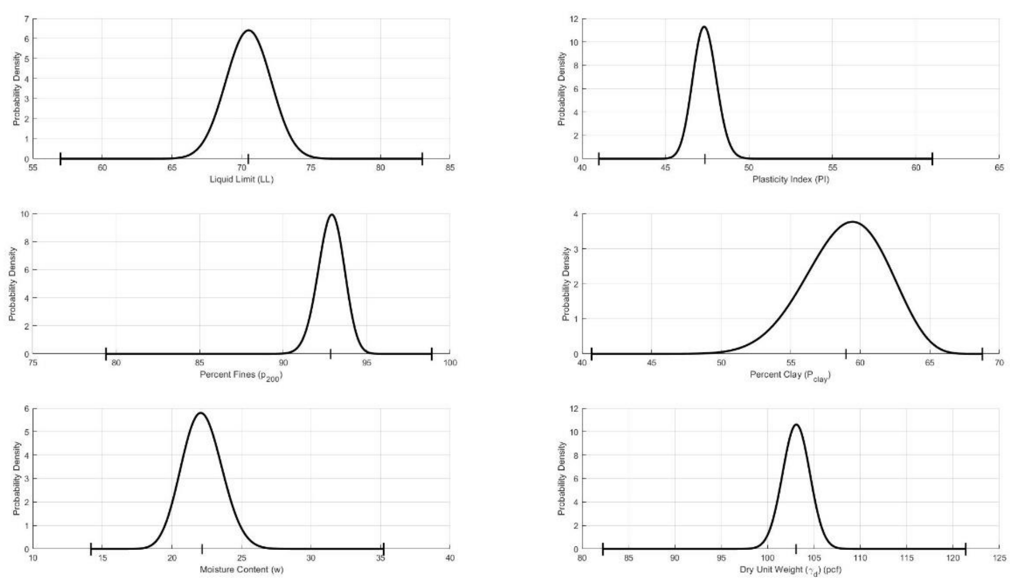


Figure 8- 21 Beta Probability Distributions for Level 2 SS with 40 ≤ wPI < 50

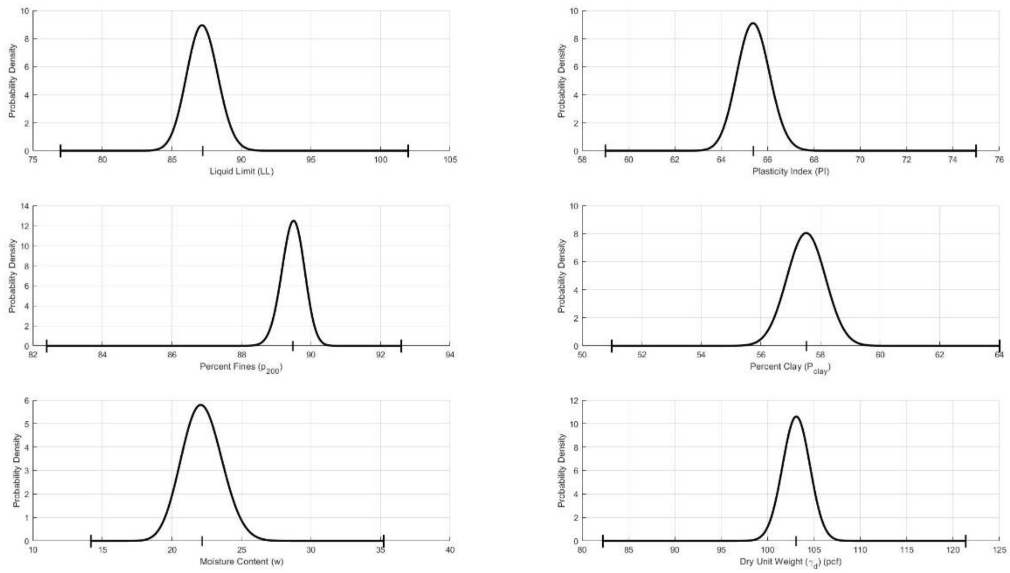


Figure 8- 22 Beta Probability Distributions for Level 2 SS with wPI ≥ 50

SS Level 3 Beta Distribution for Input Soil Properties

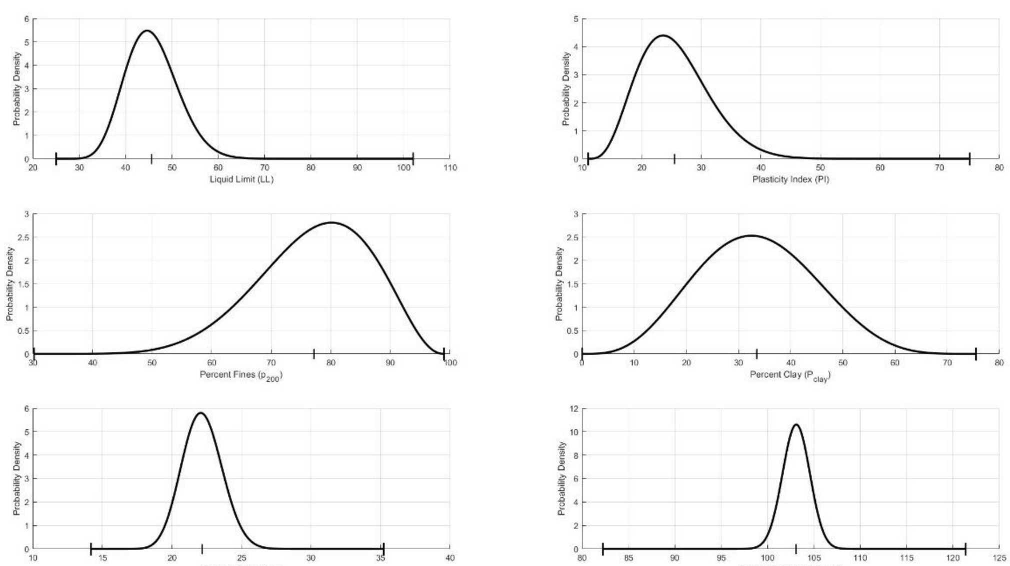


Figure 8- 23 Beta Probability Distributions for Level 3 SS with A-6 & A-7 soil with wPI>10

8.5 Randomization of Soil Index Properties

General laboratory investigations for a given project provide average values of geotechnical properties which are used as input into a deterministic solution in which only a mean value is produced. To obtain a stochastic answer, the dimensionless coefficient of variation (*CV*) is used to characterize the randomness and uncertainty in the measured properties. The coefficient of variation is generated through replicates of the test results which can require either more time and money for sampling/testing, or historical project data variance. If the coefficients of variation for the required soil properties of a project are known, sampling/testing can be reduced while increasing or maintaining the same level of confidence in the analyses and designs (provided the engineering team is experienced in statistical/stochastic analyses). As such, the coefficients of variation for the soil properties are used as key soil inputs for the stochastic volume change analyses.

Randomization of the soil properties is required for the stochastic Monte Carlo volume change analyses. Random variables are to be generated from probability distributions. Beta distributions will be used for the required soil inputs for each hierarchal level of analysis: plasticity index (*PI*), liquid limit (*LL*), percent fines/percent passing the No. 200 sieve ($P_{\#200}$), and percent $_{clay}$/percent fine than 2 microns (P_{clay}), in situ moisture content (*w*), and dry unit weight (γ_d). The LTPP soil database (FHWA) and the NCHRP 9-23 (2006) soil databases will be used to develop the statistical parameters for new subsets of soil types for the shrink-swell analysis.

It is well known that soil properties tend to be correlated within one another. Due to this high correlation between the soil properties, the random generation of each property cannot be performed individ*ually on* one another because there is a high potential that non-realistic soils will be generated. For example, the random generations of *PI*, *LL*, $P_{\#200}$, and P_{clay} separately can result in values of 55, 50, 10, and 20, respectively. It is impossible for *PI* to be greater than *LL*, and it is impossible for P_{clay} to be greater than $P_{\#200}$. Although constraints can be placed to limit these scenarios, there will s*till* be a potential for non-realistic soils to be generated. For example, the random generations of *PI*, *LL*, $P_{\#200}$, and P_{clay} can result in values of 85, 50, 10, and 5, respectively. Although these values meet the constraints of *LL* greater than *PI*, and $P_{\#200}$ greater than P_{clay}, it is highly unlikely that a natural soil with a *PI* of 50 to only have 10% fines and 5% $_{clay}$. More realistic soils can be randomly generated using regression equations for the correlated properties.

The U.S. Department of Agriculture (USDA) Natural Resources Conservation Service (NRCS) Soil Survey Laboratory (SSL) soil database was used to explore correlations and relationships between required input soil index properties: *PI*, *LL*, $P_{\#200}$, and P_{clay}. Due to significant correlation between the input soil index properties, an algorithmic approach was developed to randomly generate each property.

1.5.1 Correlation Study using NRCS Soil Database

The NRCS soil database was used to test for general correlations between the required soil index properties for the stochastic SS analysis: plasticity index (*PI*), liquid limit (*LL*), percent fines ($P_{\#200}$), and percent clay (P_{clay}). The *wPI* for each soil entry was also calculated per Zapata et al. (1999, 2000) and was included in the regression analyses. Correlations between index properties and in situ properties (density and moisture) were not explored. The NRCS soil database filtered using the following criteria in order:

- Filtered out entries with missing $P_{\#200}$ (could not classify it)
- Filtered out entries with $P_{clay} > P_{\#200}$ (non-realistic entry)
- Filtered out entries with $LL < PI$, resulted in 0 entries (non-realistic entry)
- Filtered out entries with all values the same (non-realistic entry)

- Determined AASHTO classification for each entry based on $P_{\#200}$, *PI*, and *LL*.

The total entry count after the filtering 28,323 soils.

The software Minitab (2021), version 20, was used to perform the statistical analyses described herein. Table 8- 42 presents the descriptive statistics of the explored parameters from the NRCS database.

Table 8- 42 Descriptive Statistics for Filtered NRCS Soil Data

Variable	N	N*	Mean	StdDev	Variance	CV	Minimum	Median	Maximum	Mode
p200	28323	0	56.085	25.788	665.029	45.98	0.000	57.500	100.000	85
pClay	28013	310	22.128	13.698	187.638	61.90	0.0000	21.000	85.000	22.5
pSilt	28013	310	34.084	19.746	389.902	57.93	0.000	32.500	95.500	35
LL	25264	3059	32.817	12.407	153.942	37.81	0.0000	30.000	200.000	30
PI	28274	49	10.899	9.947	98.934	91.26	0.0000	7.500	70.000	0
wPI	28274	49	7.7596	8.8222	77.8312	113.69	0.0000	4.6000	62.3000	0

The correlation coefficient was to examine relationship between the soil properties *PI*, *LL*, $P_{\#200}$, and P_{clay}. Correlations between *LL* and *PI* are expected as *PI* is determined from the difference of the *LL* and the Plastic Limit (*PL*), however, *PI* was included in the correlation evaluation, as PL is not included in the input parameters for the stochastic soil volume change analyses. The *wPI* parameter was excluded from the correlation test as it is directly calculated from *PI* and $P_{\#200}$.The Pearson correlation coefficient (*r*) was used to examine the relationship between the soil properties *PI*, *LL*, $P_{\#200}$, and P_{clay}. Figure 3-7 illustrates the different patterns in the strength and direction of the relationships between the soil properties *PI*, *LL*, $P_{\#200}$, and P_{clay} from the NRCS soil database filtered with $PI > 0$.

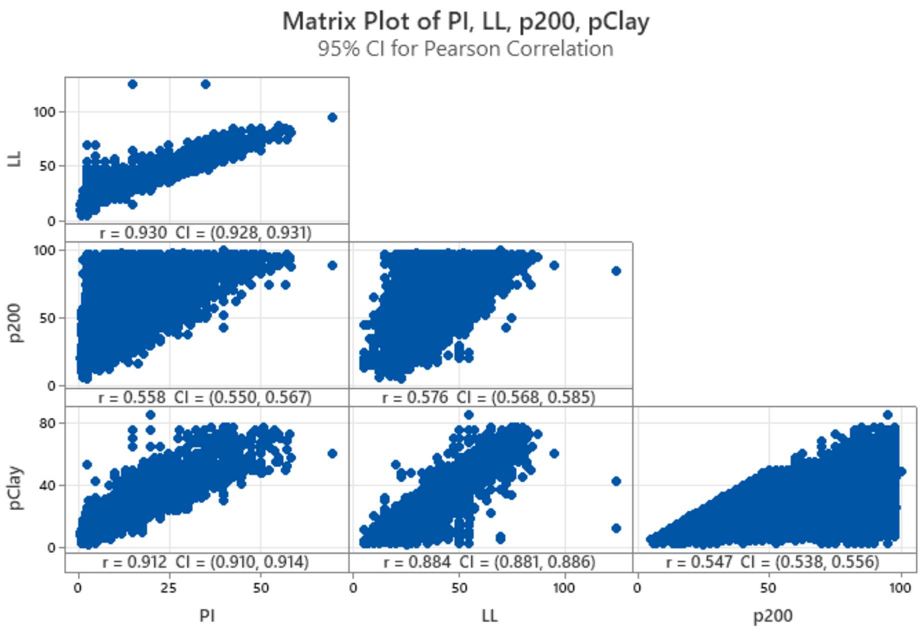


Figure 8- 24 Correlation Plots for PI, LL, P#200, and Pclay from NRCS Soil Database Filtered with PI > 0

Table 8- 43 presents the Minitab output of the correlation coefficients matrix. The Pearson (linear) correlation coefficients indicate that the *PI*, *LL*, and P_{clay} properties are highly correlated, which is expected based on the nature of the geotechnical index properties. The $P_{\#200}$ parameter resulted in a low linear correlation. The *PI* resulted in the strongest correlation to *LL* and P_{clay}, which was used as the basis of the regression study summarized in the following section.

Table 8- 43 Correlation Matrix for PI, LL, P#200, and Pclay from NRCS Soil Database Filtered with PI > 0

	PI	LL	$P_{\#200}$
LL	0.930		
$P_{\#200}$	0.558	0.576	
P_{clay}	0.912	0.884	0.547

Nine regression trails have been performed with the goal of building statistically significant relationships. (p-value < 0.1). A summary of the adjusted r-square and standard deviation of the residuals of the regression fit is presented in Table 4- 44 All of the regression relationships were determined to be statistically significant with all p-values < 0.01.

Table 8- 44 Summary of Regression Analyses for LL, PI, P#200, and P_{clay}

Trial No	Dependent Variable	Independent Variables			Linear		Quadratic (Q)/Multiple (M) [1]	
		1	2	3	R-squared (adjusted)	Residual Standard Deviation	R-squared (adjusted)	Residual Standard Deviation
1	LL	PI			86.42	4.429	86.43 (Q)	4.427
2	PI	LL			86.42	3.569	86.52 (Q)	3.556
3	P_{clay}	PI			83.15	5.310	84.17 (Q)	5.148
4	P_{200}	PI			31.18	18.693	34.71 (Q)	18.207

5	P_{200}	PI	P_{clay}				35.04 (M)	
6	P_{200}	PI	P_{clay}	LL			37.93 (M)	
7	wPI	PI			93.38	2.289	93.95 (Q)	2.187
8	wPI	PI	P_{clay}				94.07 (M)	
9	wPI	PI	P_{clay}	LL			94.25 (M)	

[1] Alternate quadratic model regression fits are automatically generated by Minitab when using the linear regression tool.

Based on the correlation and regression study, the regression models for *PI* to *LL*, *PI* to P_{clay}, and *PI* to *wPI* explained a high percentage of the variation. There was not a significant difference in the adjusted R-squared values between the linear and quadratic/multiple regression models; therefore, the linear regression models were chosen for implementation due to simplicity.

As portrayed in the correlation study, the regression models to estimate $P_{\#200}$ explained only 31.18% to 37.79% of the variation in $P_{\#200}$. Although the regressions to estimate $P_{\#200}$ indicated a statistically significant relationship ($P < 0.001$), the high scatter of residuals and relatively low r-squared does not provide high enough confidence to implement the regression models into the stochastic analyses.

8.2.
8.2.1.

1.5.2 Graphical Summary and Normality tests

The Anderson-Darling (AD) statistic is used by Minitab to measure how well the data follows a particular distribution. The null hypotheses (H_0) for the Anderson-Darling test when testing normality is that the data follows a normal distribution.

If the p-value for the Anderson-Darling test is lower than the chosen significance level of 0.05, it is concluded that the data does not follow a normal distribution (i.e., reject the null hypothesis). The graphical summaries and normality tests for each parameter are provided in Figure 8- 25 through Figure 8- 29.

Based on the results of the normality test presented in Figure 8- 25 through Figure 8- 29, the five soil properties from the NRCS soil database filtered with $PI > 0$ are not normally distributed. Based on a visual evaluation of each property histogram, Beta distributions were chosen to represent the soil properties, following the existing approach of the MEPDG.

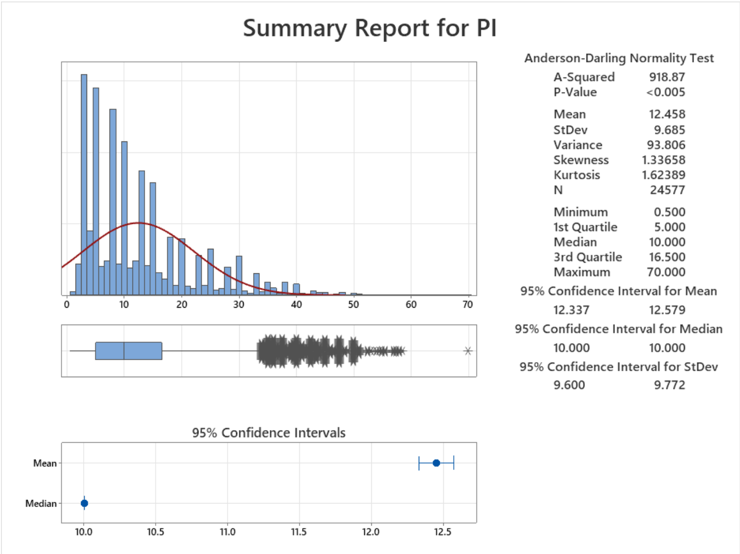


Figure 8- 25 Normality Test and Descriptive Statistics for PI from NRCS Soil Database with PI > 0

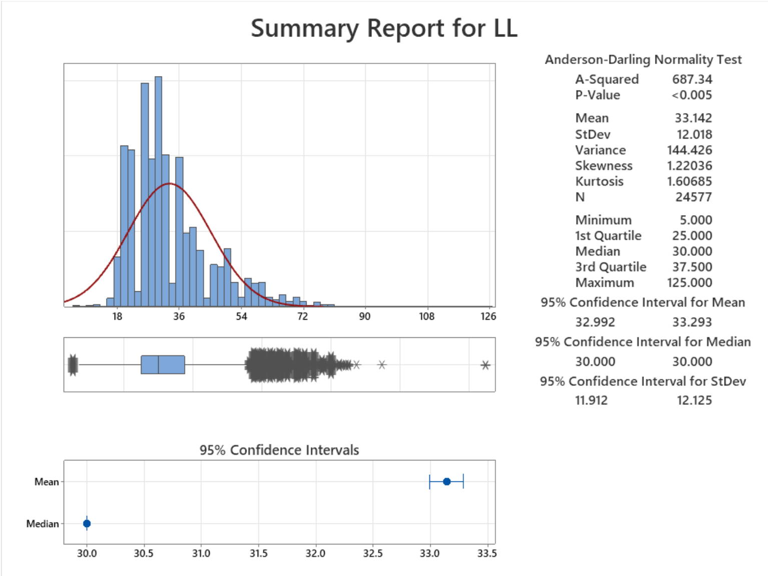


Figure 8- 26 Normality Test and Descriptive Statistics for LL from NRCS Soil Database with PI > 0

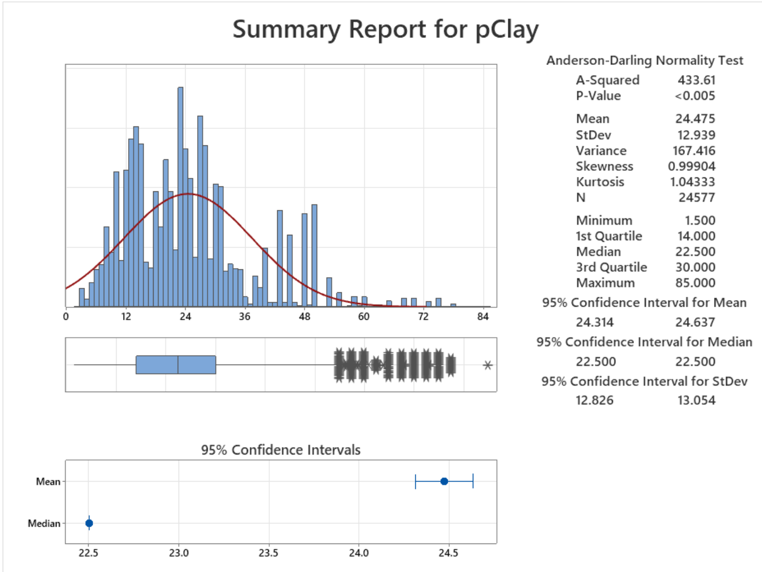


Figure 8- 27 Normality Test and Descriptive Statistics for Pclay from NRCS Soil Database with PI >0

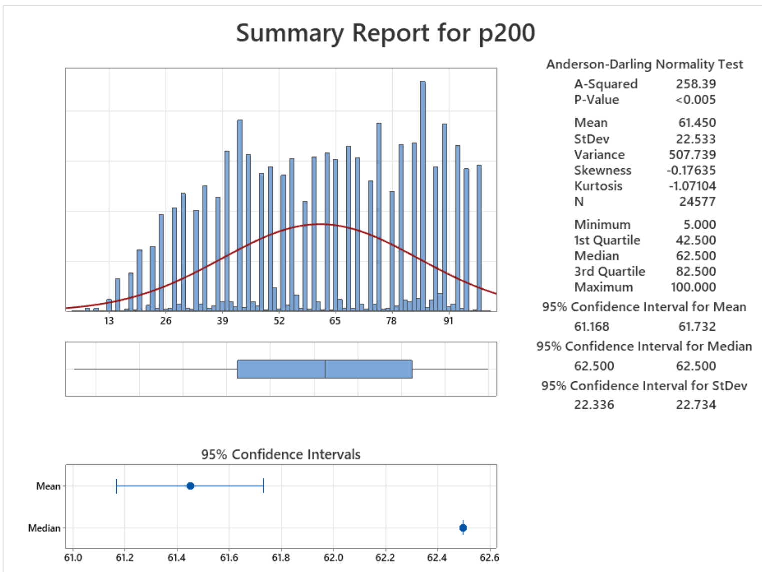


Figure 8- 28 Normality Test and Descriptive Statistics for P#200 from NRCS Soil Database with PI >0

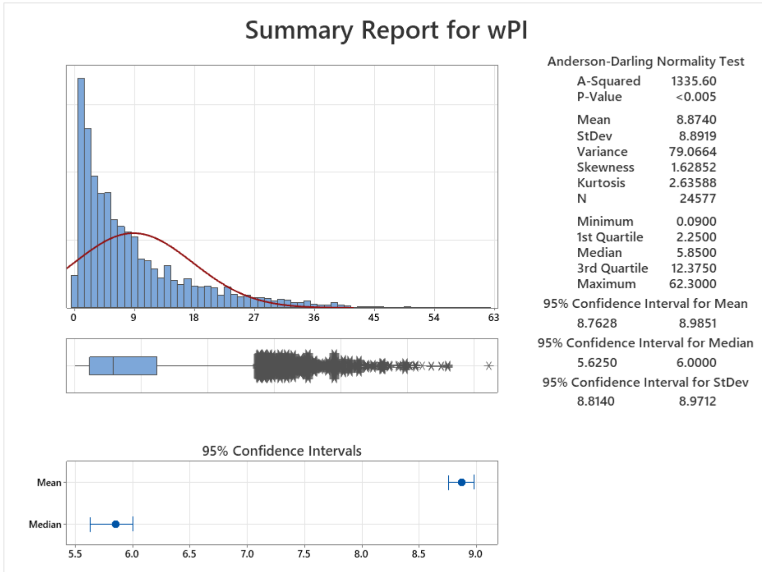
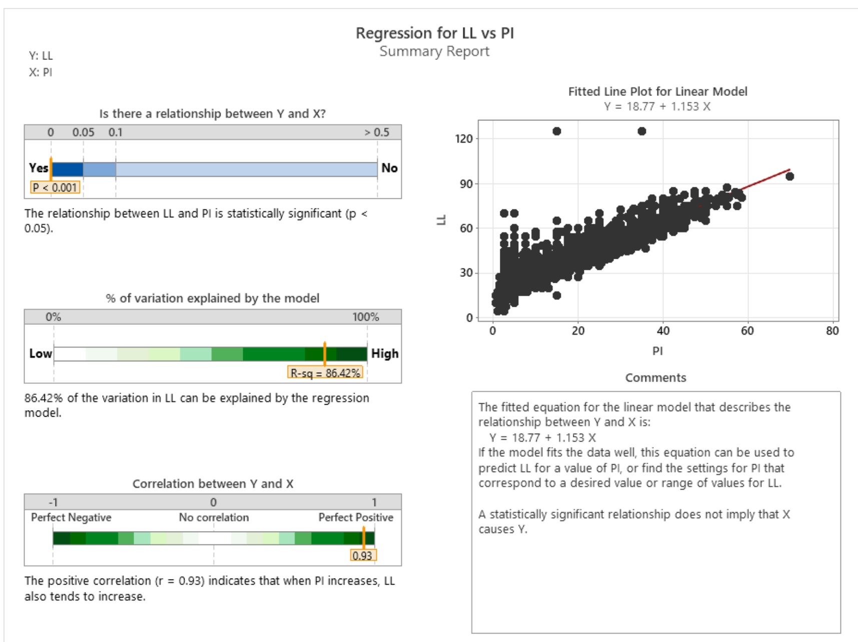


Figure 8- 29 Normality Test and Descriptive Statistics for Pclay from NRCS Soil Database with PI >0



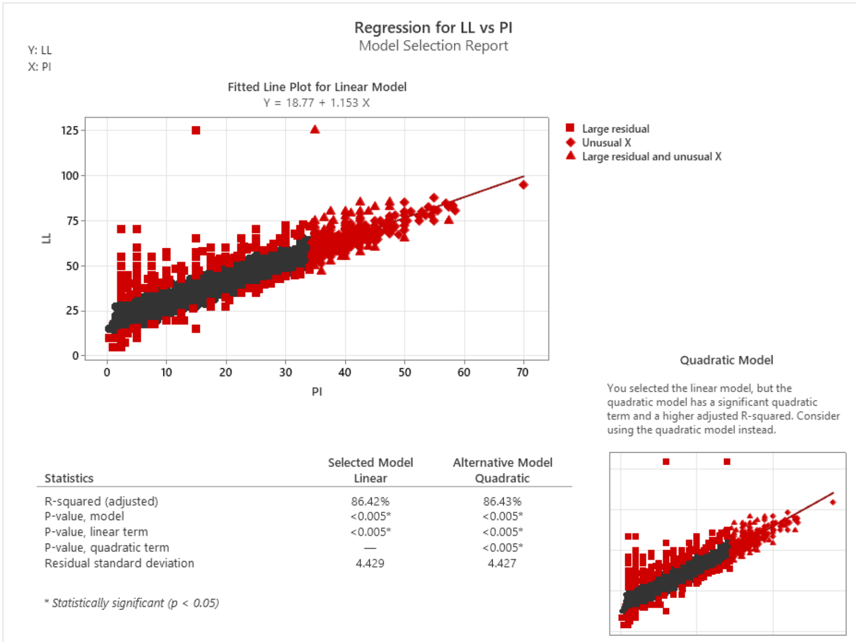
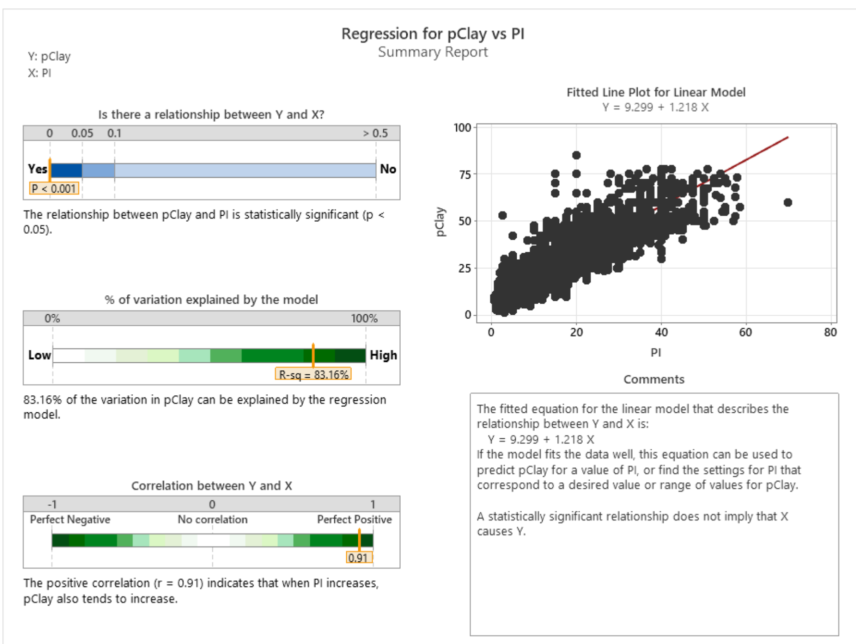


Figure 8- 30 Minitab Summary of Linear Regression of PI to LL



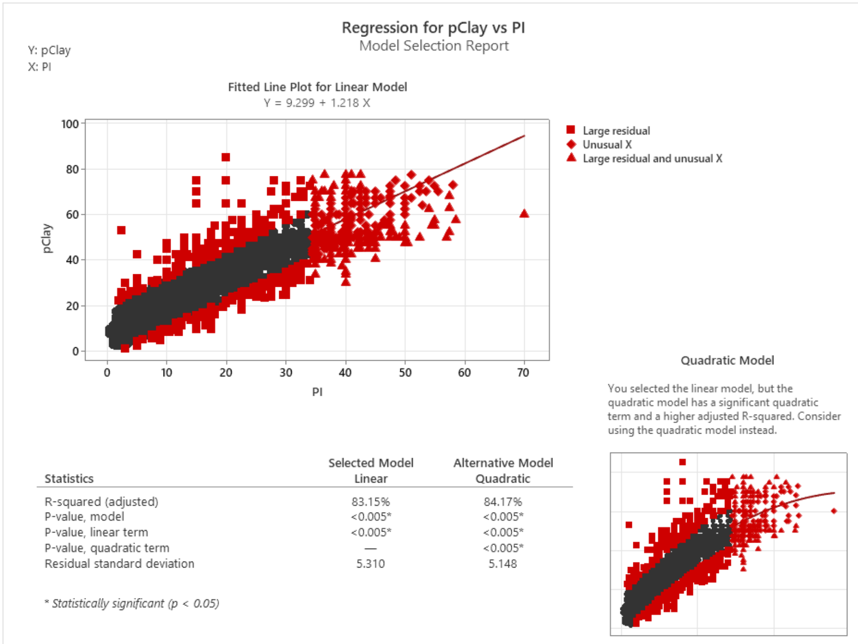
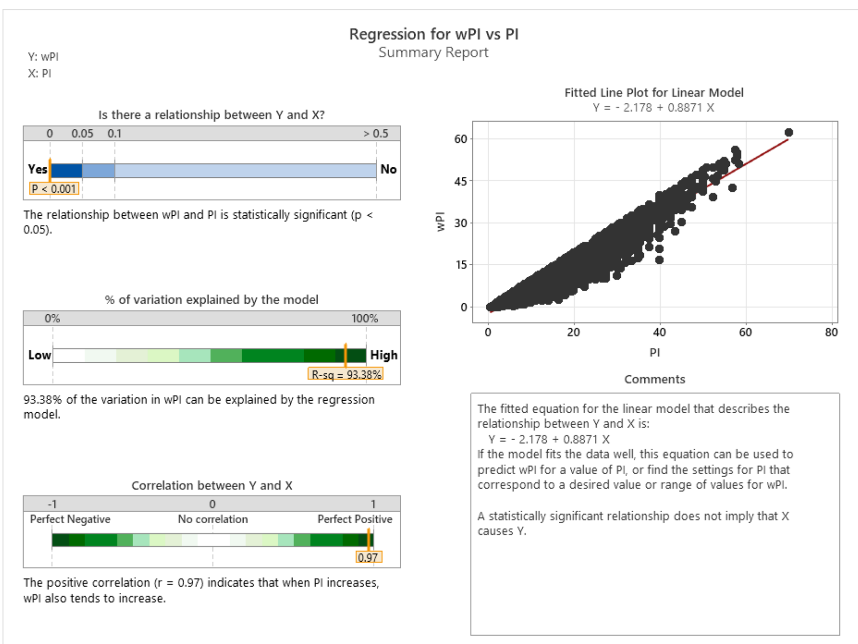


Figure 8- 31 Minitab Summary of Linear Regression of PI to Pclay



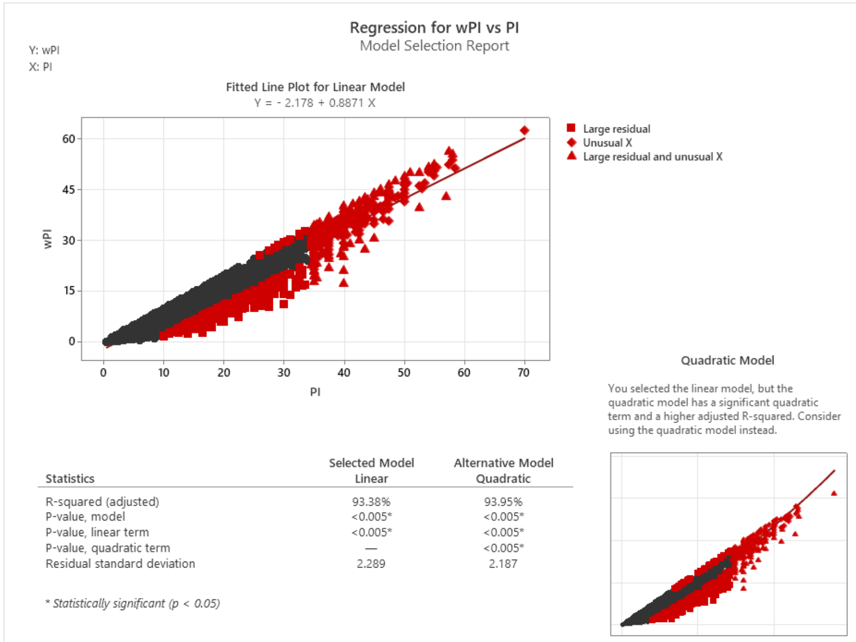


Figure 8- 32 Minitab Summary of Linear Regression of PI to wPIAlgorithm for Randomizing Correlated Soil Index Properties

To improve the randomization process to produce more natural results, an algorithmic approach was developed to randomly generate the *PI*, *LL*, $P_{\#200}$, and P_{clay} inputs for the stochastic volume change analyses using beta distributions, linear regression equations for properties correlated to *PI*, standard deviations of the residuals of the regression models, and criteria for the minimum and maximum bounds of the beta distributions based on the nature of the soil properties.

1) Randomly generated *PI* based on the <u>beta distributions</u> for the hierarchical level.
2) Randomly generated p200 based on the <u>beta distributions</u> for the hierarchical level.
3) Randomly generate *LL* based on <u>regression model</u> with *PI*
   a) Use <u>linear regression model</u> to estimate expected value of *LL* using *PI* (Eq. 8-25).
   b) Randomly generate *LL* using a beta distribution with:
      i) the mean equal to the expected value from the regression fit with *PI*,
      ii) the standard deviation equal to the standard deviation of the residuals of the regression fit,
      iii) the maximum equal to the expected value plus 3 times the standard deviation of the residuals from the regression fit, and
      iv) the minimum equal to the greater of the *PI* or the expected value minus 3 times the standard deviation of the residuals from the regression fit
4) Randomly generate P_{clay} based of regression model with *PI*
   a) Use linear regression model to estimate expected value of P_{clay} with *PI* (Eq. 8-26),
   b) Randomly generate P_{clay} using a beta distribution with:
      i) the mean equal to the expected value from the regression fit with *PI*,
      ii) the standard deviation equal to the standard deviation of the residuals of the regression fit,
      iii) the maximum equal to the expected value plus 3 times the standard deviation of the residuals from the regression fit, and
      iv) the minimum equal to the greater of zero or the expected value minus 3 times the standard deviation of the residuals from the regression fit

Note that the engineering soil properties of in situ moisture content and dry unit weight are also required as inputs for the stochastic analyses but it assumed that these engineering properties act independently and are not correlated to any of the of the soil index properties or to one another. As such, the moisture content and dry unit weight can be randomly generated from the beta distributions without correlation concerns, described further herein.

The normal distribution is not bound between any values (i.e., the left tail of the distribution will approach negative infinity and the right tail of the distribution will approach positive infinity). As such, issues can arise when generating random numbers for data sets which represent percentages or index values that must be greater than 1 because there is a possibility of producing negative values. This scenario is applicable to the required soil index properties of PI, LL, p200, and pClay as each parameter is a percentage greater than 0%. The PI and LL parameters do not have an upper bound value but p200 and pClay cannot be greater than 100%. Furthermore, PI is computed from LL and pClay is a fraction of p200 (i.e., must be less than), resulting in high correlation between the parameters, which is discussed further herein. Albeit these limitations for application in stochastic analyses with natural soil properties, the normal distribution is still used as effective tool for preliminary screening of normality for input variable and residuals of regression fits.

Randomization of the soil properties is required for the stochastic Monte Carlo volume change analyses. To validate that the Beta random number generator function in MATLAB, histograms with increasing numbers of Monte Carlo simulations (*nMC*) can be generated and the predetermined Beta distributions for each input parameter can be visually checked for fit to the distribution, as shown in the example in Figure 8- 33.

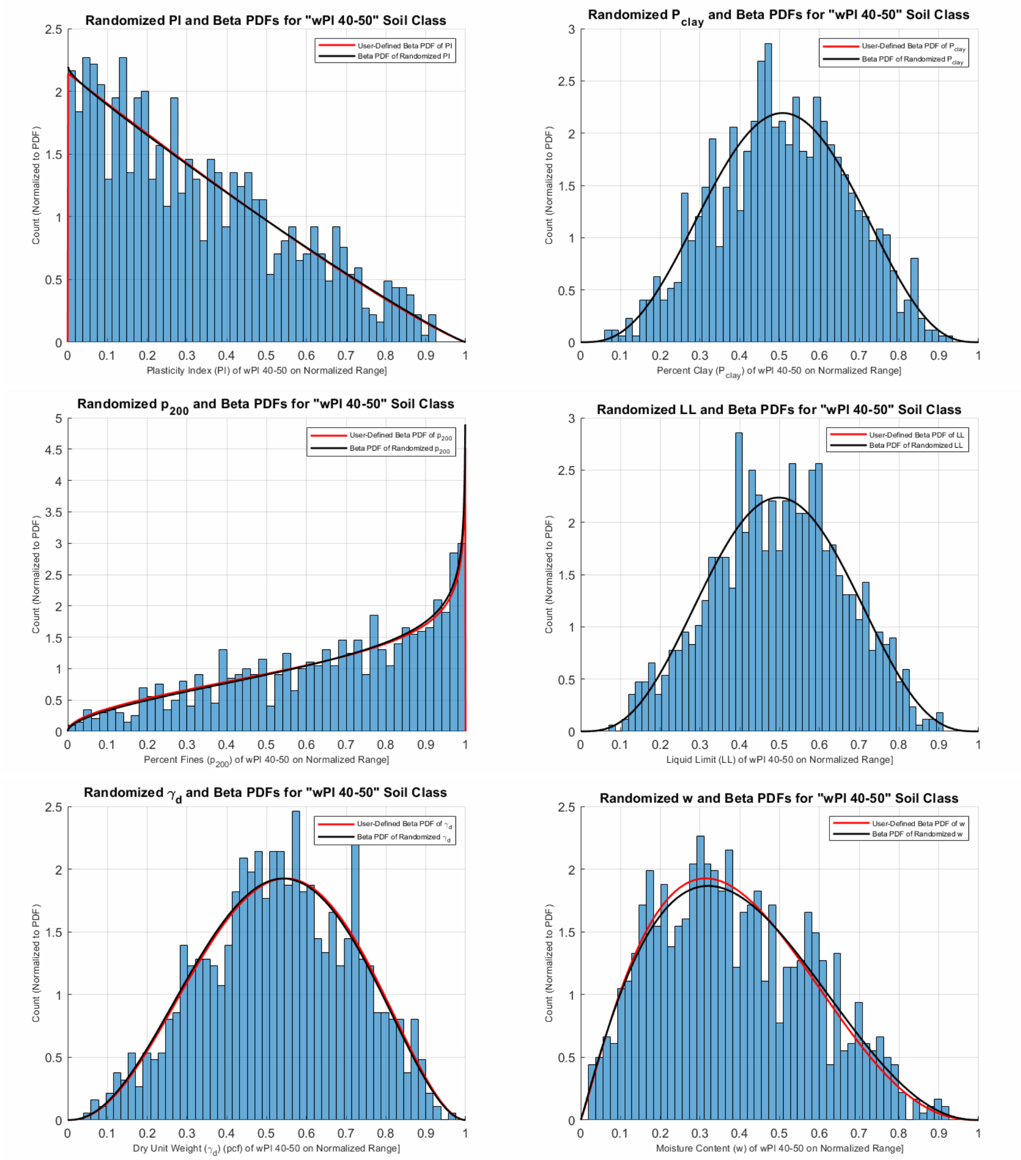


Figure 8- 33 Example of Histograms and Fitted Beta Distributions for Randomly Generated Data Points PI, p200, pClay, LL, d_UW, and w.

8.6 Implementation of Random Soil Properties Generator

The hierarchical descriptive statistics with Beta distributions and the random soil properties algorithm are being considered for implementation in the Pavement Mechanistic-Empirical Design (PMED) guide, also referred to as MEPDG, as part of the NCHRP Project 01-59. As part of the work on that project, the author developed a computer program with a graphical user interface which allows users to perform most of the stochastic analyses presented herein. Figure 8- 34 through Figure 4- 42 present excerpts (screenshots) from the developed software program which provide an example of the random soil index property model using a few different scenarios.

Figure 8- 34 presents the default descriptive statistics for the Level 3 “Clayey FGM” material soil group. Note that only the Mean, coefficient of variation (*CV*), minimum (Min), and maximum (Max) are displayed. The standard deviation and variance parameters are not necessary as they can be represented using *CV*, and the Beta shape factors

were deemed by the author and the NCHRP 01-59 research group to not be allowed for direct user updating. Calculations of Beta shape factors are generally based on known statistical moments (mean and variance) and known/desired limits of the distribution (min and max). It is uncommon for the beta shape factors to be the known parameters requiring the statistical moments and limits of the distributions to be back calculated. As such, the program automates the calculation of the updated Beta shape factors/distributions based on the user input of either Mean, *CV*, Min, or Max. Not that the work "either" is not a mistake in the previous statement, as the develop random soil properties model uses a Bayesian type framework to update the default Beta Distribution based on any known (user-input) parameters and does not need all four to be input.

Design Level 3 ▼ SS/FH FH ▼ Soil Type Clayey FGM ▼

	Mean	CV	Min	Max
p200	75.7493	0.2103	36.1000	100.0000
PI	21.2993	0.4277	10.5000	75.0000
wc	20.3036	0.2406	10.1300	35.2100
d_UW	105.9969	0.0845	82.2400	126.1100

Figure 8- 34 Default Descriptive Statistics for the Level 3 "Clayey FGM" Material Soil Group

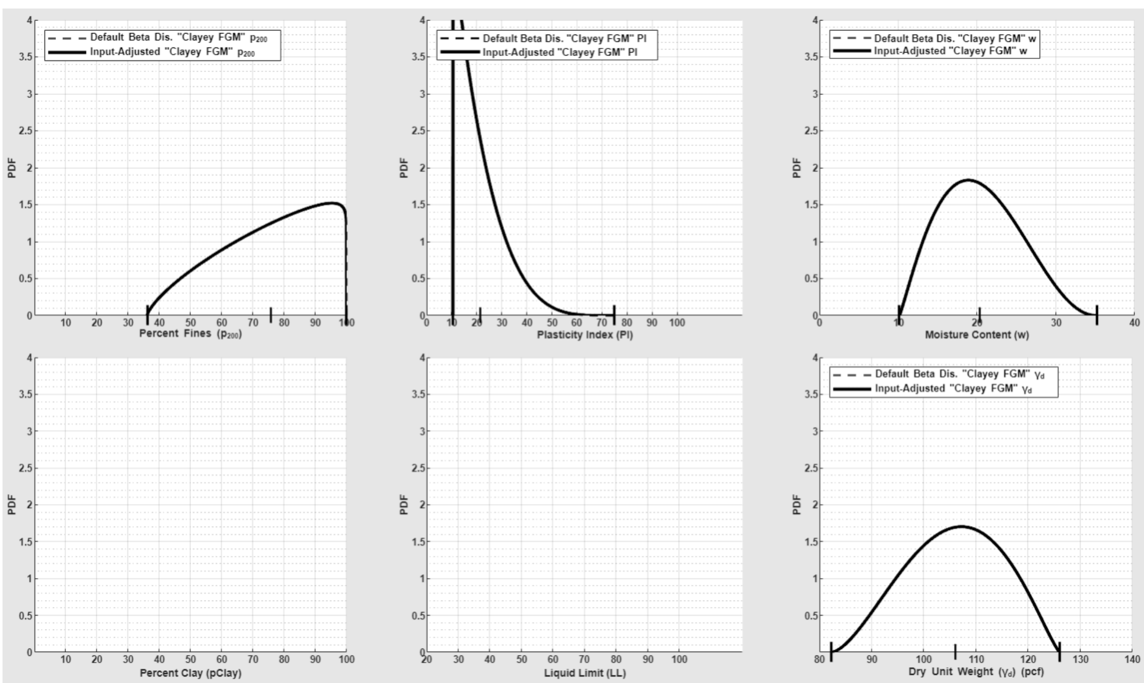


Figure 8- 35 Default Beta Distributions for the Level 3 "Clayey FGM" Soil Group

The Beta distribution for the input soil properties percent fines (*p200*), *Plasticity Index* (*PI*), moisture content (*w*), and dry unit weight (*d_UW*) are displayed in Figure 8- 35. Note that units for the dry unit weight is in pounds per cubic foot (pcf), and the other three variables are in percent (%). The Beta Distributions of the additional parameters *LL* and *pClay*, are not defined by user input due to the high correlation with *p200* and *PI*, but rather produced using the regression equations and algorithmic process proposed herein. Figure 8- 36 presents the Beta Distributions for *LL* and *pClay* generated by fitting the histograms of the data produced using the randomized regression-based algorithm (although the actual histogram is not displayed on this portion of the interface). The number of randomized draws used to produce the distributions in Figure 8- 36 was 1000.

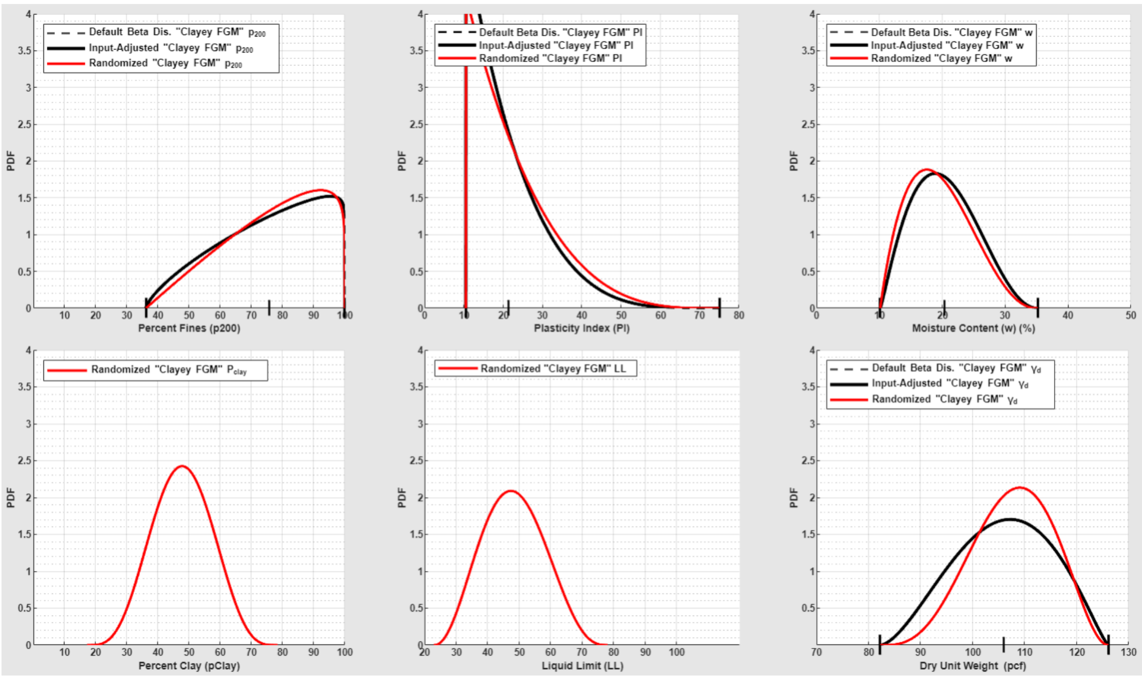


Figure 8- 36 Default and Randomized Beta Distributions for the Level 2 "Clayey FGM" Soil Group using 1000 Monte Carlo draws

This number of draws was chosen to present these examples as it generally provides a relatively good fit to the default distributions but not the most ideal fit. One can expect visually noticeable differences between the default distributions and the distributions fit to the randomly generated data when the number of draws is at or below 1000. Generally, the number of draws is governed by the stability of the produced variance of the complete Monte Carlo analysis, which for the full shrink-swell volume change model presented in this study, the required number of simulations (draws) will need to be 10,000 at minimum, which is discussed further herein. As such, the randomized soil property algorithm will generally produce stable representations of the variability of the soil input properties.

The difference between the variability of the default hierarchical soil groups can be visually evaluated using Figure 8- 37 and Figure 8- 38 which present the default descriptive statistics and randomized distributions for the Level 2 A-7-6 soil which encompasses clayey soils as presented in the previous example of Level 3 "Clayey FGM" in Figure 8-36. Note that there is a difference in the x-axis scale for the Beta distributions for the different soil groups. An example of the randomly generate Beta Distributions for the proposed soil groups for Shrink-Swell soils is presented in Figure 8- 38 and Figure 8- 40.

Design Level: 1 or 2 | SS/FH: FH | Soil Type: A-7-6

	Mean	CV	Min	Max
p200	81.7784	0.1565	37.5000	98.0000
PI	28.7988	0.2722	14.0000	75.0000
wc	20.3036	0.2406	10.1300	35.2100
d_UW	105.9969	0.0845	82.2400	126.1100

Figure 8- 37 Default Descriptive Statistics for the Level 2 A-7-6 Soil Group

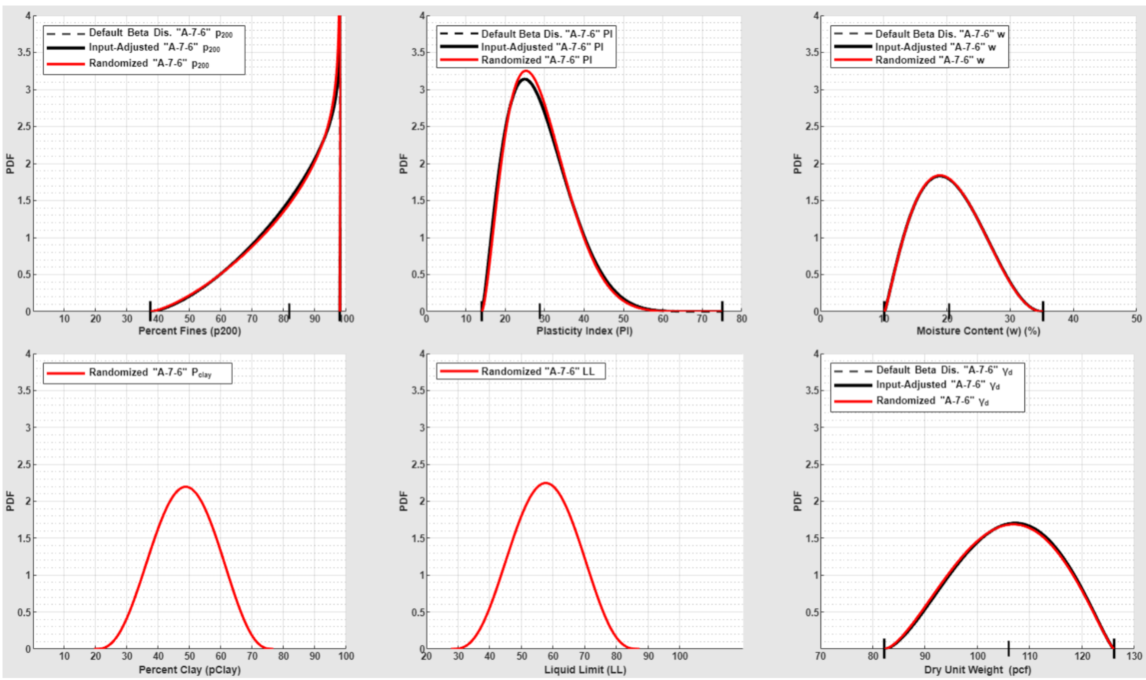


Figure 8- 38 Default and Randomized Beta Distributions for the Level 2 A-7-6 Soil Group using 1000 Monte Carlo draws

Design Level: 1 or 2 | SS/FH: SS | Soil Type: wPI 30-40

	Mean	CV	Min	Max
p200	88.7400	0.1077	60.6000	99.0000
PI	39.1100	0.1561	31.0000	61.0000
wc	22.1800	0.1786	14.1900	35.2100
d_UW	103.0900	0.0703	82.2400	121.3900

Figure 8- 39 Default Descriptive Statistics for the Level 2 "wPI 30-40" Soil Group

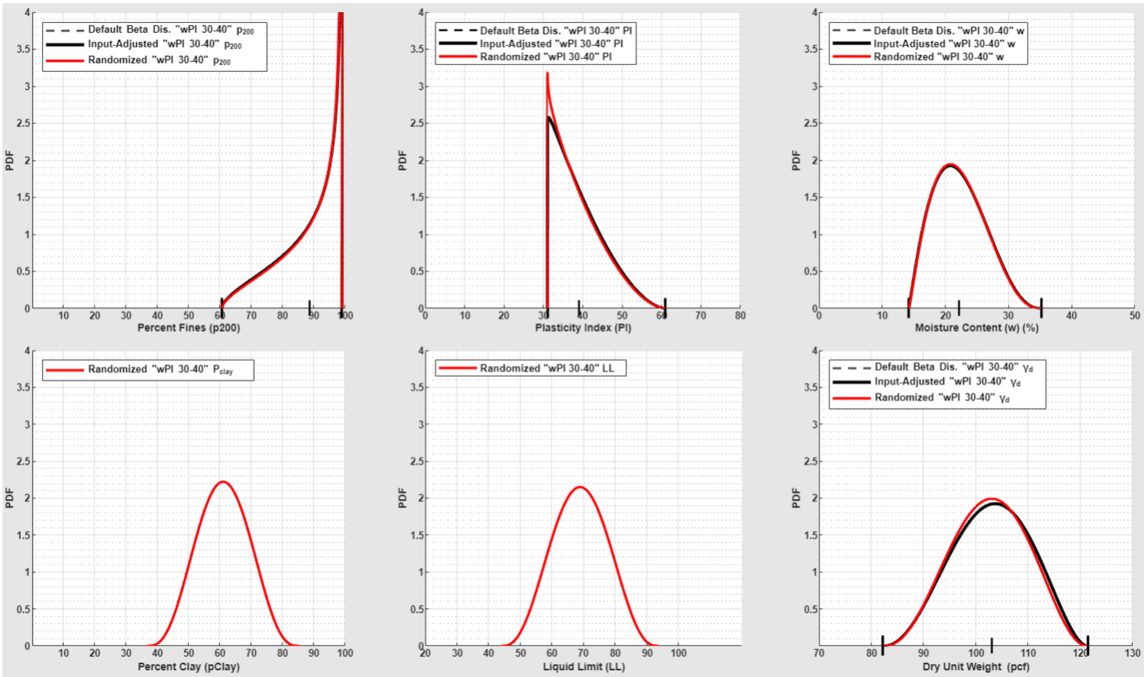
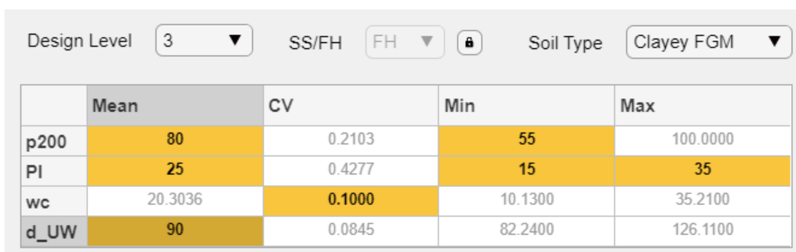


Figure 8- 40 Default and Randomized Beta Distributions for the Level 2 "wPI 30-40" Soil Group using 1000 Monte Carlo draws

The computer program developed by the author allows for the user to adjust any of the input descriptive statistics for the four input soil properties. This allows for the user to include any site-specific knowledge into the generation of the Beta distribution, which can either decrease or increase the overall variability of the parameter compared to the default soil group values. Figure 8- 41 presents an example of user adjusted parameters (highlighted cells) for various, but not all, of the descriptive statistics of the input variables.


Design Level 3 SS/FH FH Soil Type Clayey FGM

	Mean	CV	Min	Max
p200	80	0.2103	55	100.0000
PI	25	0.4277	15	35
wc	20.3036	0.1000	10.1300	35.2100
d_UW	90	0.0845	82.2400	126.1100

Figure 8- 41 Example of Various User Adjusted Descriptive Statistics for the Level 3 "Clayey FGM" Soil Group

The program displays the default Beta Distribution along with the user defined Beta distribution and the randomized beta distribution based on the user adjusted descriptive statistics to allow for a visual comparison, as shown in the Figure 8- 42 example.

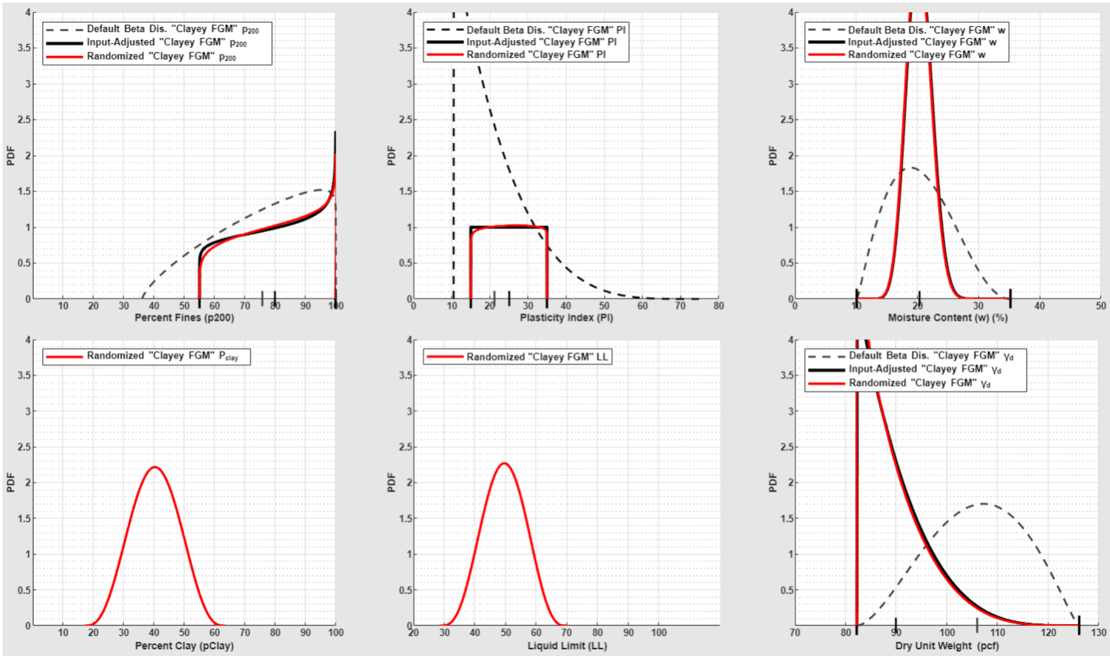


Figure 8- 42 Example of the Randomized Beta Distributions (red) based on the various User Adjusted descriptive statistics (Figure 8- 41) for the Level 3 "Clayey FGM" Soil Group

8.7 Implementation of Random Soil Properties Algorithm in Practice

Randomization of the soil properties is required for the stochastic Monte Carlo volume change analyses. Random variables are generated from probability distributions. Beta distributions were generated for the required soil inputs for each hierarchal level of analysis: plasticity index (*PI*), liquid limit (*LL*), percent fines/percent passing the No. 200 sieve ($P_{\#200}$), and percent $_{clay}$/percent fine than 2 microns (P_{clay}), in situ moisture content (*w*), and dry unit weight (γ_d). The LTPP soil database (FHWA) and the NCHRP 9-23 (2006) soil databases were used to develop the statistical parameters for new subsets of soil types for the shrink-swell analysis.

This study introduced an updated approach to stochastically model the variability of the required soil properties. A database of updated statistical parameters for common soil index properties has been compiled for the AASHTO soil groups and for soils groups differentiated by wPI. A Bayesian framework for randomly generating natural combinations of highly correlated variables was developed. Adjustment of the datasets used for the hierarchical levels of the descriptive statistics to better represent the common soil types susceptible to shrink-swell potential. The ability for the engineer to use historical/prior data as a starting point to represent the variability of common soil properties provides a tool which can be used for preliminary sensitivity analyses prior to a site visit, which can provide insight to which soil properties need additional measurements via sampling and testing. Improvement to the overall level of confidence in the geotechnical produced output, and associated variability, can be increased using any site-specific adjustment to the default Beta distributions.

1.7.1 Limitations

The limitations pertaining to the Bayesian characterization model for generating randomized inputs of common soil properties should be understood prior to consideration of implementation into engineering practice:

- The NRCS and LTTP databases were used to generate descriptive statistics for the default variability characterization for the defined soil groups. Although these databases are considered by the field to adequately represent most soil types, there is always a chance for a location to consist of soils which exhibit characteristics away from the norm. It is always recommended that some site-specific data be obtained to gain an understanding of the material types at hand and rule out any potential unusual scenarios.

- The proposed approach for characterizing the variability of common soil properties uses the general variability of the measured properties for a given soil type to represent the 2D variability at the subject site. As such, the proposed framework can be considered a pseudo-2D approach for characterizing soil variability but does not include the modern techniques of quantifying spatial variability.

8.8 References

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