Project No. 01-59

THE DETERMINISTIC MODEL USED TO EVALUATE FROST DEPTH AND FROST HEAVE OF PAVEMENT

APPENDIX 6

THE SIMPLIFIED 1-D FROST HEAVE MODEL

MAY 2023

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6. Introduction

In order to evaluate the frost behavior of soil impact on pavement surfaces, a 1-D model was proposed. The model objectives, background, theory, and calculation example are discussed in this Appendix. The outputs of the 1-D model include the predicted frost depth (FD) variation with time, the predicted thawing depth (TD) variation with time, the predicted frozen length (FL) variation with time and the frost heave (caused by water expansion and segregation potential) variation with time. It utilizes climate data as inputs and can simulate both freezing and thawing time series processes.

6.1 Objectives

The targets of the study in this chapter are summarized below:
1. Develop a procedure only using simple temperature related term as a boundary condition for predicting frost depth in uncovered or pavement covered areas.
2.Evaluate time-series frost penetration based on a weak coupled hydro-thermal 1-D model that considers pavement structure, thermal properties, and in-situ moisture conditions.
3.Propose correlation procedures for calculating segregation potential of soils without performing subgrade soil experiment.
4. Use the proposed 1-D model to compute the segregation potential induced frost heave by integrate the estimated frost penetration and segregation potential.
5.Via post processing of the solved 1-D model results to obtain thawing penetration, frozen length, and frost heave due.
6.Solve 1-D model using finite element modeling method by software COMSOL
7.Code the 1-D model in MATLAB following COMSOL calculation and post processing steps.
8.Extend the coded MATLAB 1-D model to a user interface that can perform level-based Monto Carlo analysis

6.2 Relative background

6.3 The simplified 1-D model theory

6.3.1 The governing equation

The new model is a weak-coupling 1-D model. The energy migration in porous media was described by the modified Fourier's equation as shown below:

$$C_a \frac{\partial T}{\partial t} = \nabla(\lambda \nabla T) \quad \textbf{(6-1)}$$

where C_a is the apparent heat capacity; T is the temperature; t is time; λ is the thermal conductivity. In the above equation, the methods used to determine C_a and λ depend on the design level selection. In most situations, the thermal properties are associated with water content. Hence, part of the impact of hydraulic filed can be incorporated into the analysis and make the model weak thermal hydro coupled. In addition, the weak coupling is also achieved by taking the effects of groundwater table (GWT) into account, because the GWT elevation change with time can indirectly reflect the hydraulic influence of rainfall on soil water content with time. To simulate the phase change process, an equivalent heat capacity, equal to the latent heat divided by a small temperature range around 273.15K, is used to represent the effect of latent heat.

6.3.2 Model geometry

The 1-D model geometry is composed by a group of continuous segments along a unique direction with total length of 15ft (around 4.5m). The length of the model is determined by calibrations and set as default value of the model. In the model geometry, the different segments represent different pavement layers. The example of a four-segment 1-D model is shown in **Error! Reference source not found.**. The example geometry includes surface layer, base layer, subbase layer and subgrade layer.

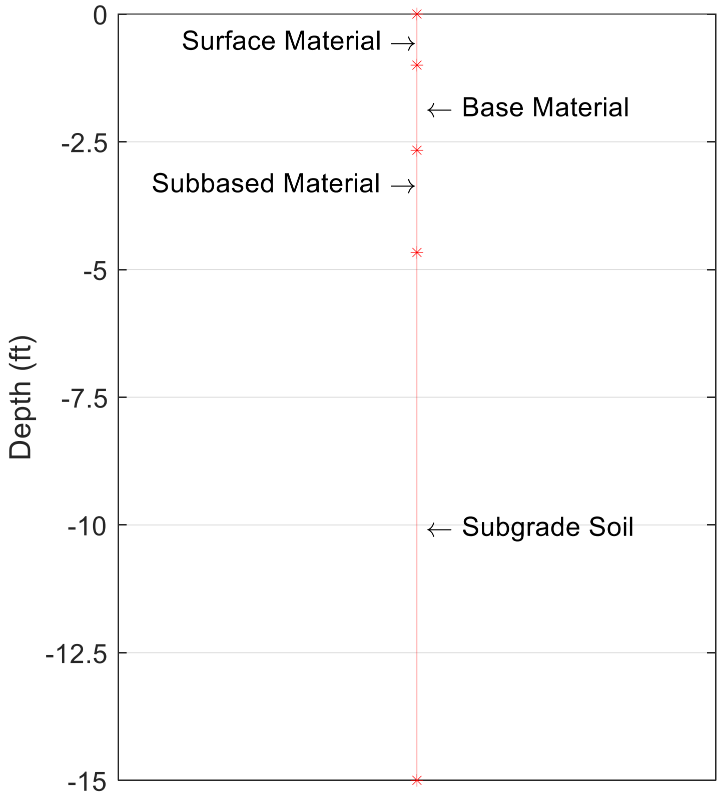


Figure 6-1 The example of a four-segment 1-D mode geometry for a four-layer pavement section

6.3.3 Model boundary conditions

Through assigning Neuman boundary conditions, the model considered the influence of the ambient temperature. To be specific, the heat influx induced by ambient temperature variation was incorporated by Newton's law of cooling:

$$n \cdot (\lambda \nabla T) = h_c(T_{amb} - T) \tag{6-2}$$

where n is the normal unit vector of the boundary; λ is the thermal conductivity, T is the temperature at boundaries; h_c is the heat transfer parameter, and T_{amb} is the ambient temperature. Note that here h_c is a calibrated parameter, the calibration detailed was presented in Appendix 7. The ambient temperature heat influx boundary was added on the top boundary (point) of the 1-D model.

The lower boundary is a thermal insulation boundary where heat flux always equals to 0. According to observation of the site-measured ground temperature data from SMP, it was found that the variation of temperature gradient at a given depth with different time and site location will overall decreases with depth. As a result, the heat variation of heat flux will also decrease with depth, when the depth is15ft (or 4.5m), such heat flux is close to 0. Hence, the default model geometry is set as 15 ft and the lower boundary is set as a thermal insulated boundary.

6.3.4 Model inputs and correlations

The model inputs depend on the hierarchical level of design as summarized in **Error! Reference source not found.** and explained in detail in the following paragraphs. Level 1 will include parameters that will require experimental measurements, Level 2 include parameters that can be obtained by empirical formulation or correlation with other properties; and Level 3 comprises default values resulting from national and local calibration efforts.

Table 6-1The three level inputs of the simplified 1-D model

Climatic inputs	Level 1	Level 2	Level 3
• Latitude and longitude of the site, used to determine the average monthly air temperature	√	√	√
Geological inputs			
• Monthly groundwater table depth	√	X (Needs freezing and non- freezing season GWT)	X (Needs freezing and non- freezing season GWT)
Structure and materials inputs			
• Pavement layer geometry	√	√	√
• Layer material type or soil classification	X	√	X
• Physical properties of layers: gradation, percent of fine content, percent of silt, Atterberg limits	√	√	√
• Specific gravity of solids	√	X Default values	X Default values
• Dry unit weight	√	√	X Default values
• Gravimetric water content at the beginning of the freezing season simulation	√	√	√
• Initial soil temperature profile	√	X	X
• Thermal properties: thermal conductivity and heat capacity of pavement surface materials	√	X Thermal conductivity: Johansen (1975) method; Heat capacity: USACE correlation equations	X Thermal conductivity: Johansen (1975) method; Heat capacity: USACE correlation equations

√: Provided by the user, X: Not needed from the user

6.3.4.1 Climatic inputs

For Levels 1, 2 and 3 design, the historical monthly average air temperature inputs are necessary. This can be provided by the closest weather station. To access the weather station data, latitude and longitude are needed. The model uses moving weighted average method to forecast temperature based on historical temperature data. It is recommended that the historical data starts from at least 20 years before the start of the design construction for any level design, otherwise the temperature prediction model may have convergency problems. For details about the temperature prediction model, please refer to Appendix 9.

6.3.4.2 Ground water table

Ground water table (GWT) is needed for all levels design due to its obvious influence on frost heave in engineering practice. Level 1 design needs monthly varied GWT based on historical data. Level 2 and 3 only need GWT of freezing and non-freezing seasons. In the 1-D model, the GWT elevation data was applied as a time-dependent interpolation function. Using this function, the model can control the transient soil saturation condition accordingly. To be specific, when the soil is below the GWT elevation, it is assumed the soil is saturated, where the volumetric water content equals the soil porosity at that depth. When the soil is above the GWT elevation, the volumetric water contents are determined by the user defined layer water content input.

6.3.4.3 Pavement layer geometry

Pavement layer geometry needs to specify the number of layers and the thickness of each layer, which is necessary for any design level. In the 1-D model, different layers are assigned different layer numbers. Upper layer usually has larger layer number. The default layer number of the lowest layer is #1, which must be the subgrade layer. The top layer is usually the asphalt concrete layer during design.

6.3.4.4 Layer material type or soil classification

For level 2 design, layer material type or soil classification needs to be clarified during setting the model input, because the material type is necessary for evaluating thermal conductivity in level 2 design. Since level 1 design defines thermal conductivity by user and level 3 design uses default material types for the layers, these two levels do not need to provide material type information.

6.3.4.5 Gradation, Atterberg limits and Segregation potential

Segregation potential (SP) is an important parameter for the 1-D model to estimate segregation caused frost heave. For all the three design levels, the same methods were used to evaluate SP. Given that most of the layers above subgrade layers are barely frost susceptible, the 1-D model only consider the segregation potential of subgrade layer.

According to Konrad (2005) the following equation is used to calculate segregation potential at zero pressure SP_0:

$$\mathrm{SP}_0 = \frac{[116 - 75 log d_{50}(FF)] \times 10^3 mm^4/(℃ \cdot s \cdot g)}{S_s} \quad \textbf{(6-3)}$$

Where S_s is soil specific area and d_{50}(FF) is the average grain size (in μm) of the fines fraction. Since SP_0, S_s, and d_{50}(FF) are not usually being experimentally measured, correlations plus assumptions were applied to evaluate their values, hence, they were not required for any of the three levels. For all design levels, the 1-D model used site corrected segregation potential to calculate the frost heave:

$$\mathrm{SP}_{field} = SP_0 \exp\,(-aP_e) \quad \textbf{(6-4)}$$

where P_e is the overburden pressure calculated by considering the unit weight and layer length above the soil; a is function of $d_{50}(FF)$ can be estimated by method proposed by Konrad (2005):

$$a = 5[d_{50}(FF)]^{0.45} \quad \textbf{(6-5)}$$

The LTPP form TST_SS02_UG03 presents gradation analysis and diameters range of gravel, coarse sand, fine sand, silt, clay, and colloids, as shown in **Error! Reference source not found.**. Note that the effective diameter of each type of particle size was assumed to be the middle value of the given diameter range, as shown in **Error! Reference source not found.**. The d_{50}(FF) parameter was calculated using the weighted effective diameter of silt, clay, and colloids when their percentage were given in the three design levels. The effective diameter of silt, clay, and colloids used were 0.038mm, 0.0015mm, and 0.0005mm as shown in Table 6-3. The weights are the percentage of silt, clay, and colloids found from the LTPP form TST_SS02_UG03.

Table 6-2 LTPP data form TST_SS02_UG03 indicated soil particle diameter range

LTPP form name	Table head	Table head meaning	Table head description
TST_SS02_UG03	GT_2MM	Percent Greater Than 2 mm	Percent of particles larger than 2 mm.
TST_SS02_UG03	COARSE_SAND	Coarse Sand	Percent of coarse sand size particles (2 - 0.42 mm).
TST_SS02_UG03	FINE_SAND	Fine Sand	Percent of fine sand size particles (.42 - .074 mm).
TST_SS02_UG03	SILT	Silt	Percent of particles between .074 - .002 mm.
TST_SS02_UG03	CLAY	Clay	Percent of clay size particles (.002 mm).
TST_SS02_UG03	COLLOIDS	Colloids	Percent of colloid size particles (.001 mm).

Table 6-3 Effective diameter and specific surface area (with sphere particle) of different particles

Particle	Effective particle diameter(cm)	Specific surface area (mm^2 g^-1)
Gravel	0.2	1100.11
Coarse sand	0.121	1800.37
Fine sand	0.0247	8900.97
Silt	0.0038	61700.28
Clay	0.00015	1481400.81
Colloids	0.00005	4444400.44

The specific surface area is correlated to the proportion of the different particle size groups (sand, clay, and silt). The following equation is used to calculate S_s:

$$\mathrm{S_s} = (-6.56 + 3.96\ \mathrm{Clay\%}) * \mathrm{Clay\%} + (227.28 - 3.298\ \mathrm{Sand\%}) * \mathrm{Sand\%} + (359.23 - 5.59\ \mathrm{Silt\%}) * \mathrm{Silt\%} \quad \textbf{(6-6)}$$

where Clay%, Sand%, and Silt% are the percentage of clay, sand, and silt among these three size groups. This equation was proposed based on Ersahin et al. (2006).

6.3.4.6 Specific gravity and dry unit weight

Level 1 design requires user defined specific gravity of soil solids. Through several trials, it was found that the specific gravity (G_s) has insignificant influence on frost heave results. Hence, level 2 and level 3 design use default and constant specific gravity of solids for all layers with $G_s = 2.7$. This value is the average G_s among the collected LTPP soil data for more than 500 sections of different layers. In the 1-D model, G_s is mainly used to evaluate material porosity by equation (6-7) as shown below.

Level 1 and level 2 design require user-defined dry unit weight (γ_d) for each layer, including the pavement surface material. For all the three levels, the γ_d is used to evaluate the segregation potential and soil porosity by the following mass-volume relationship:

$$n = 1 - \frac{\gamma_d}{G_s \gamma_w} \quad \textbf{(6-7)}$$

Where γ_d is the dry unit weight of the soil, γ_w is the unit weight of water and G_s is the specific gravity of the solids. Specifically, in level 2 and level 3 design, γ_d is also used for estimating the thermal conductivity and volumetric heat capacity (see details in 6.3.4.9). For level 3 design, the detailed default G_s and γ_d information is summarized in Table 6-4.

Table 6-4 Default G_s and γ_d for different layers in level 3 design

Layer type	Default parameters	Values	Data Source
Asphalt Concrete	Dry unit weight, γ_d	150 pcf	Default value obtained from the software Pavement ME design
base/subbase	Dry unit weight, γ_d	129.81 pcf	Average subgrade dry unit weight of 156 LTPP sections from TST_UG07_SS07_A
Subgrade	Dry unit weight, γ_d	105.91 pcf	Average subgrade dry unit weight of 359 LTPP sections from TST_UG07_SS07_B
base/subbase	Specific gravity, G_s	2.70	Average G_s of 3591 sets of base/subbase sample from LTPP
Subgrade	Specific gravity, G_s	2.70	Average G_s of 3484 sets of base/subbase sample from LTPP

6.3.4.7 Gravimetric water content at the beginning of the freezing season simulation

The initial water content listed in Table 6-1 is the water content profile at the beginning of the freezing season. This value can significantly influence the predicted frost heave. At Level 1 analysis, the water content profile can be input by the user or can be estimated by the Enhanced Integrated Climatic Model (EICM) within the current AASHTOW are software for Levels 2 and 3.

6.3.4.8 Initial soil temperature profile

The initial soil temperature profile is requested for level 1. For Level 2 and Level 3 analyses, the initial temperature will be predicted based on the harmonic function presented below (Doughty et al., 1991):

$$T(z,t) = \mathrm{T_m} + T_a e^{-z\sqrt{\frac{\omega}{2a_s}}} \cos\left[\omega(t - t_0) - z\sqrt{\frac{\omega}{2a_s}}\right] \quad \textbf{(6-8)}$$

where $\mathrm{T_m}$ is the annual ambient temperature at the ground surface; $\mathrm{T_a}$ is the amplitude of temperature variation at the ground surface; z is depth; t is time; $\omega = 2\pi/\tau$ is the angular frequency of temperature variations, where τ is the period of the variation; $\mathrm{a_s}$ is the solid thermal diffusivity; $\mathrm{t_0}$ is the maximum surface temperature occurs. While using this function, the 1-D model will provide estimated values of $\mathrm{T_m}$, T_a, $\mathrm{t_0}$, and τ based on historical monthly temperature data. The $\mathrm{a_s}$ will be evaluated based on thermal properties of the soil which can be calculated via correlation equations as presented in 6.3.4.9. The model can then generate initial temperature profiles (z vs. T) accordingly.

6.3.4.9 Thermal properties: thermal conductivity and heat capacity of pavement surface materials and the soil's solid phases

The thermal conductivity of pavement materials is requested for Level 1 design for all pavement layers. According to literature review, there are different ways to evaluate thermal conductivity of soils, e.g., USACE correlation charts (1988), Johansen (1978), Côté and Konrad (2005), According to (Farouki 1981),the ohansen (1978) method is more suitable and simpler to calculate the thermal conductivity of the soil in cold regions. It (combined with Côté and Konrad, 2005) was taken as the default method to evaluate the thermal conductivity of the layers below surface material for both level 2 and 3 analyses：

$$\lambda = \left(\lambda_{sat} - \lambda_{dry}\right)K_e + \lambda_{dry} \tag{6-9}$$

where, λ_{dry} and λ_{sat} are the thermal conductivity of dry and saturated soils in W/m/K, respectively; and K_e is the normalized thermal conductivity (also called Kersten number). A semi-empirical relationship is used to estimate λ_{dry}:

$$\lambda_{dry} = \frac{0.135\rho_b + 64.7}{2700 - 0.947\rho_b} \tag{6-10}$$

where, ρ_b is the bulk density of soil in kg/m^3 and the number 2700 in equation (6-9) is the soil solids density in kg/m^3. The geometric mean equation is used to evaluate λ_{sat}:

$$\lambda_{sat} = \lambda_s^{1-n}\lambda_w^n \tag{6-11}$$

where, λ_w is the thermal conductivity of water in W/m/K (a default value of 0.594 W/m/K was used in the model); n is the porosity of the soil; and λ_s is the thermal conductivity of soil solids, which is determined using another geometric mean equation about quartz content of the total solids:

$$\lambda_s = \lambda_q^q\lambda_o^{1-q} \tag{6-12}$$

where, q is the quartz content of the total solids content; λ_q is the quartz thermal conductivity in W/m/K (using default value of 7.7 W/m/K in model); and λ_o is the thermal conductivity of other minerals taken as 2.0 W/m/K for soils with $q > 0.2$, and 3.0 W/m/K for soils with $q \leq 0.2$. Since the amount of quartz content is difficult to obtain from the user, it is assumed that the quartz content equals the sand content of the solids, as assumed by Lu et al. (2007). The Kersten number K_e is evaluated by Côté and Konrad (2005), as follows:

$$K_e = \frac{kS_r}{1 + (k-1)S_r} \tag{6-13}$$

where, k is an empirical parameter and S_r is the degree of saturation. The k values used by Côté and Konrad (2005) were 4.60, 3.55, 1.90, and 0.60 for gravel and coarse sand, medium and fine sand, silty and clayey soils, and organic fibrous soils, respectively. It can be found that the thermal conductivity only depends on the porosity, gradation, and degree of saturation in the above method. Gradation is a required input for the 1D model. The layer porosity as well as the degree of saturation can be evaluated using certain required inputs presented in Table 6-1 in the model. Therefore, specific thermal conductivity of layers is not the required input in levels 2 and 3.

The volumetric heat capacity is required from the user in Level 1 design for all pavement layers. For level 2 and 3, the volumetric heat capacity of layers below surface material will be calculated with the following correlations (Joint Departments of the Army and Air Force USA 1988):

$$C_u = \gamma_d(c + \frac{w}{100}) \quad \textbf{(6-14)}$$

$$C_f = \gamma_d(c + \frac{0.5w}{100}) \quad \textbf{(6-15)}$$

where C_u and C_f are the volumetric heat capacity for unfrozen and frozen soil, respectively; γ_d is dry unit weight; c is the specific heat of the soil solids (0.17 for most soils); and w is the water content of soil in percent of dry weight.

For pavement surface material, the default thermal properties are utilized in both level 2 and 3 design. The default values which are used by the MEPDG software are shown in Table 6-5.

Table 6-5 The default pavement surface thermal parameters for level 2 and 3 design

Surface type	Heat capacity (BTU/lb-degF)	Thermal conductivity (BTU/hr-ft-degF)
Flexible pavement	0.23	0.67
JPCP	0.28	1.25
CRCP	0.28	1.25

6.3.5 Frost depth and heave evaluation

Through solving the governing equation (6-1) combined with all used defined inputs as well as model bulit-in correlations, the model can obtain temperature variation along depth with time. The position where the temperature equal to 32°F is viewed as frost depth (FD) in the 1-D model during freezing season. While during thawing season, there might be two positions showing 32°F temperature along the model geometry. The upper one is defined as thawing depth and the lower one is defined as frost depth. Note that during freezing season, the theoretical thawing depth is 0ft. The depth difference of the frost depth and thawing depth is defined as frozen length.

The frost heave is calculated by considering the volume change of water frozen inside pore space and the volume change due to ice segregation (ice lenses):

$$\Delta h_{total} = \Delta h_s + \Delta h_i \quad \textbf{(6-16)}$$

$$\Delta h_s = 1.09 * SP_{field} * \nabla \mathrm{T} * \Delta \mathrm{t} \quad \textbf{(6-17)}$$

$$\nabla h_i = 0.09 * n * FD \quad \textbf{(6-18)}$$

where, Δh_{total} is total frost heave; Δh_s is frost heave due to water intake (or segregation potential); Δh_i is frost heave due to in-situ pore water freezing; $\Delta \mathrm{t}$ is time interval; *n* is soil porosity; *FD* is frost depth; SP_{field} is the site corrected segregation potential; and $\nabla \mathrm{T}$ is the temperature gradient in the frozen fringe. In equation (6-17), the 1-D model uses equation (6-5) to calculate SP_{field} and $\nabla \mathrm{T}$ is assumed to be the temperature gradient at the model simulated frost front. In equation (6-18), FD is the model simulated frost depth.

6.4 Solving the 1-D model: result examples

In essence, solving the 1-D model is solving the governing partial differential equation (6-1). However, it is not straightforward to get the solution due to the high nonlinearity caused by a lot of time and space dependent parameters in model (e.g., thermal conductivity, heat capacity, and ground water table). Hence, the FEM software COMSOL is first used to solve the 1-D model. COMSOL uses finite element method to solve PDE and has strong power to handle non-linear problems. The COMSOL solved 1-D model result example was presented in Figure 6-1.

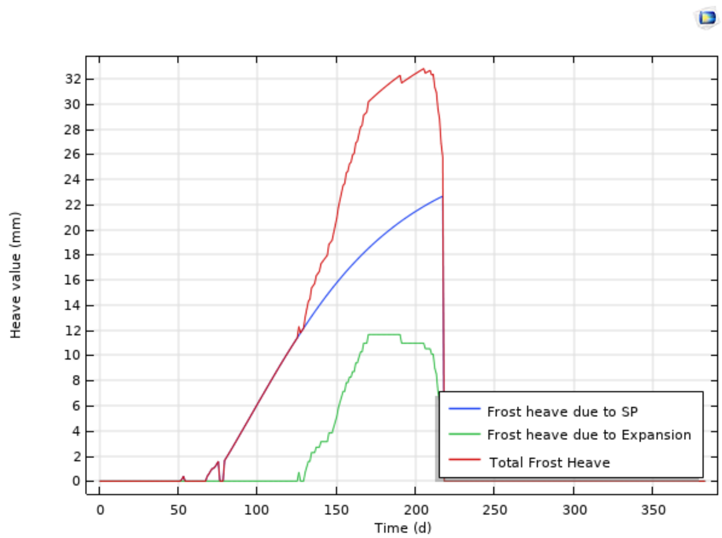
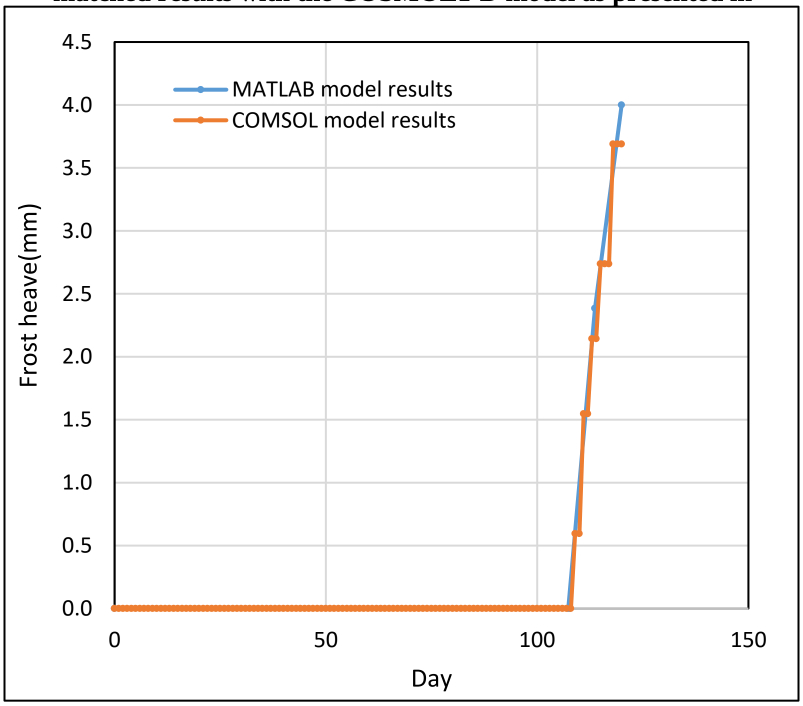


Figure 6-2 Example: the 1-D model evaluated frost heave vs. days solved by COMSOL (for LTPP sections Minnesota 1018)

Following the setting, solving and post-processing steps, the 1-D model was programmed in MATLAB. Like COMSOL, the MATALB 1-D model support user-defined input, mesh as well as time step setting, PDE solving, and result post processing. The MATLAB 1-D model showed overall consistent and well-

matched results with the COSMOL1-D model as presented in



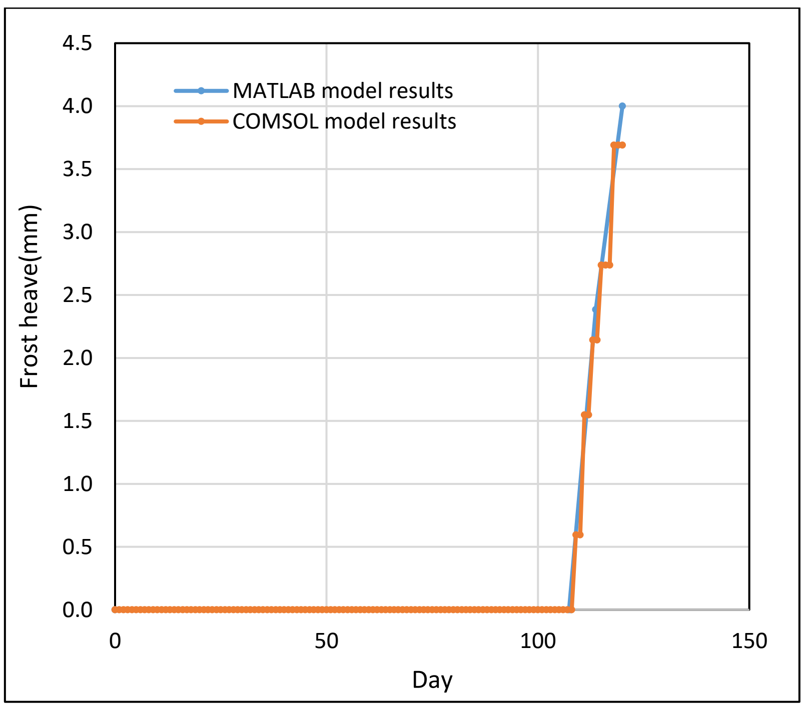


Figure 6-4Figure 6-3 and

Figure 6-4. The example of the MATALB 1-D model general results is shown in

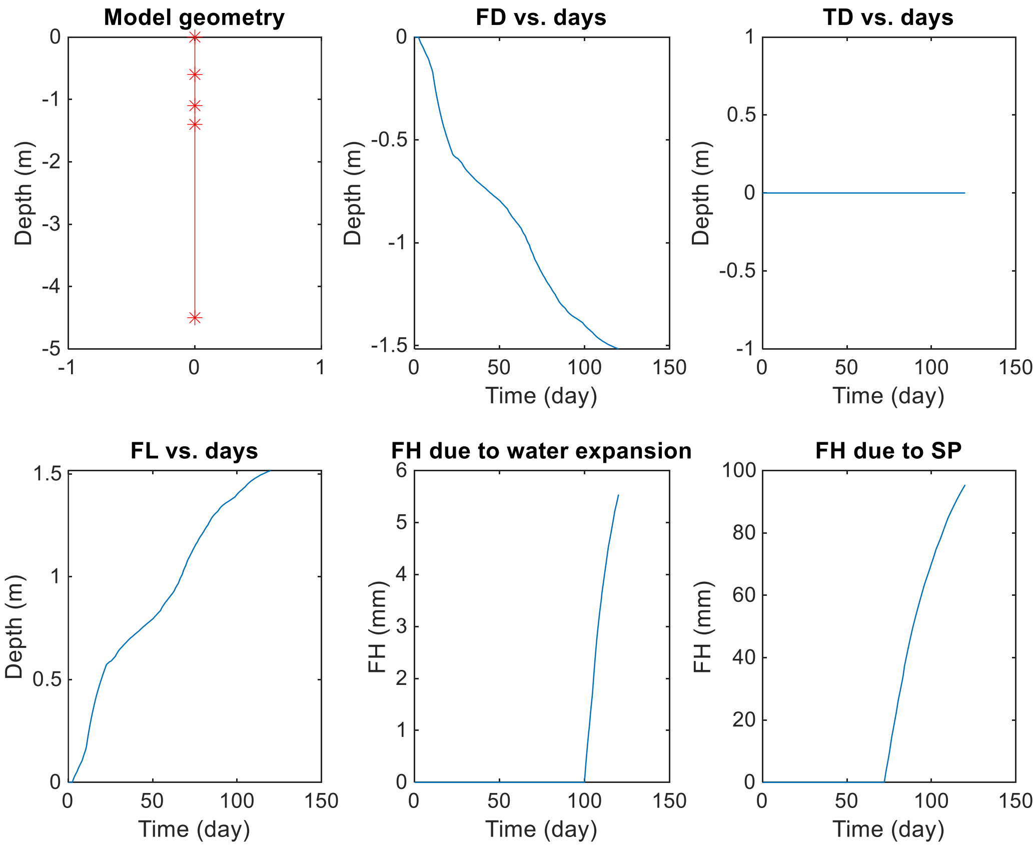


Figure 6-5. As presented in

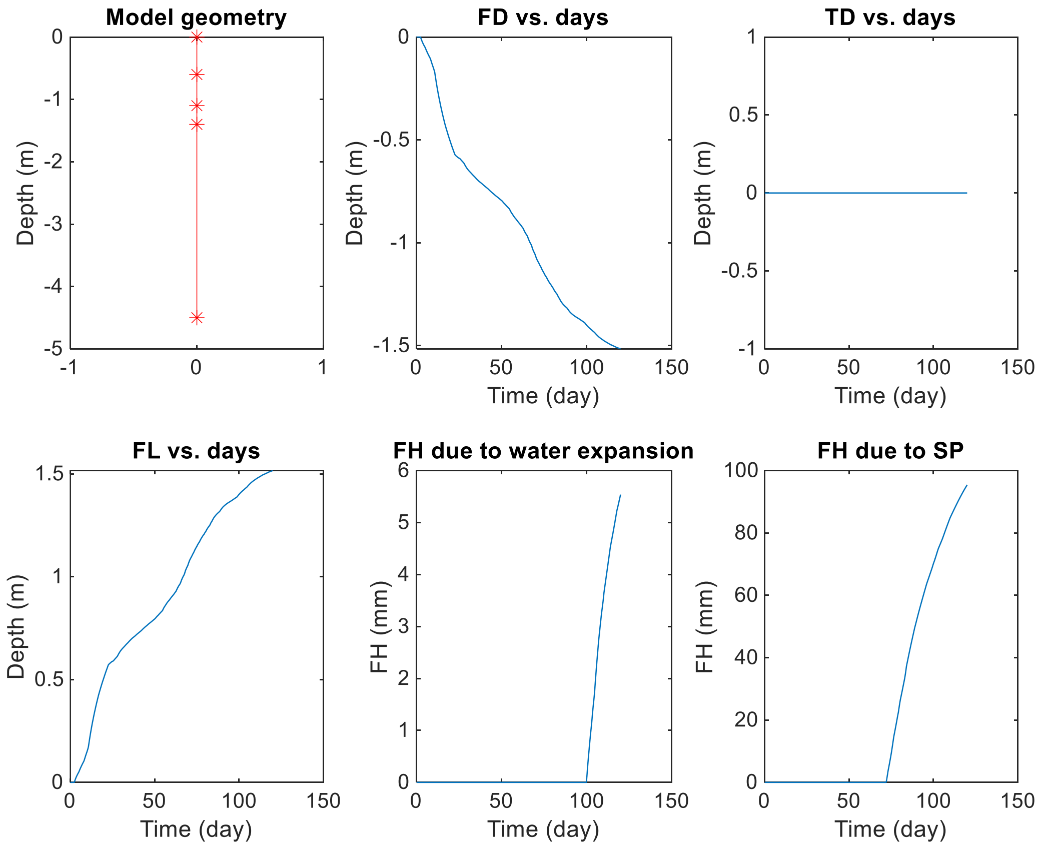


Figure 6-5, results of model geometry, frost depth (FD), thawing depth (TD), frozen length (FL), Frsot heave (FH) due to water expansion, and FH due to segregation potential (SP) were plotted. Results of Figure 6-3 to

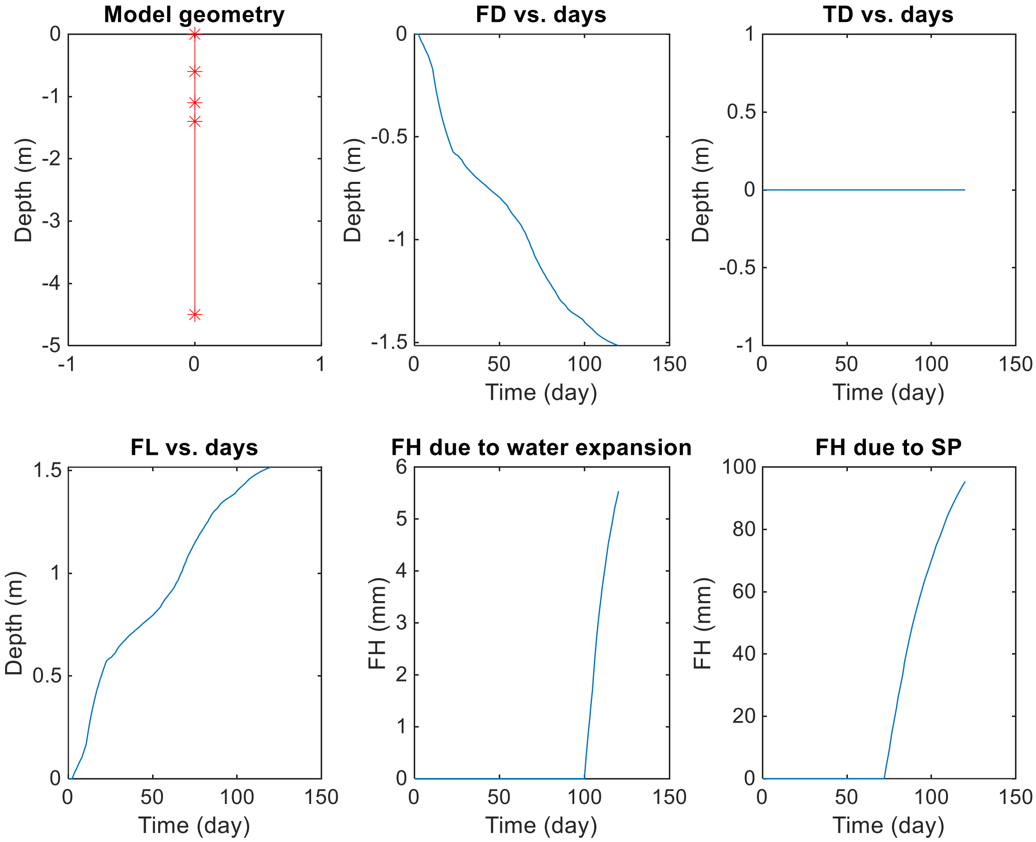


Figure 6-5 were obtained from the Finland site case analysis for model calibration and verification. Details are presetned in Appendix 7.

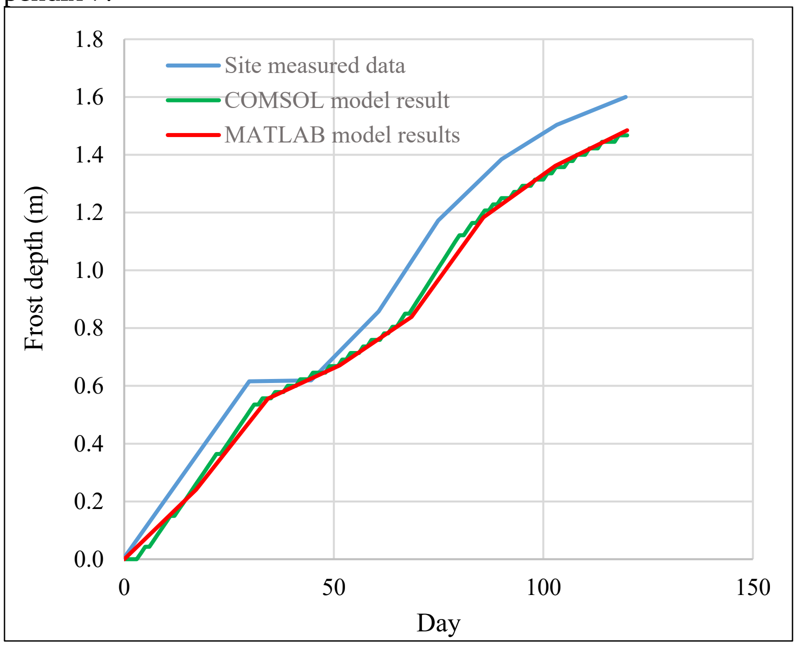


Figure 6-3 FD prediction comparison between the MATLAB 1-D and COMSOL 1-D model (used data from Finland site at Joensuu P33)

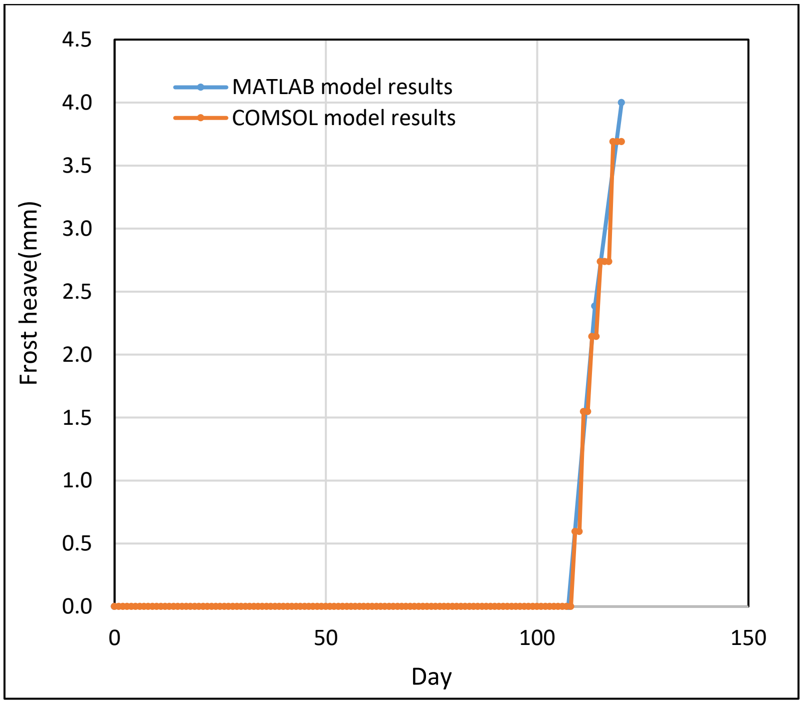


Figure 6-4 Water expansion FH prediction comparison between the MATLAB 1-D and COMSOL 1-D model (used data from Finland site at Joensuu P33)

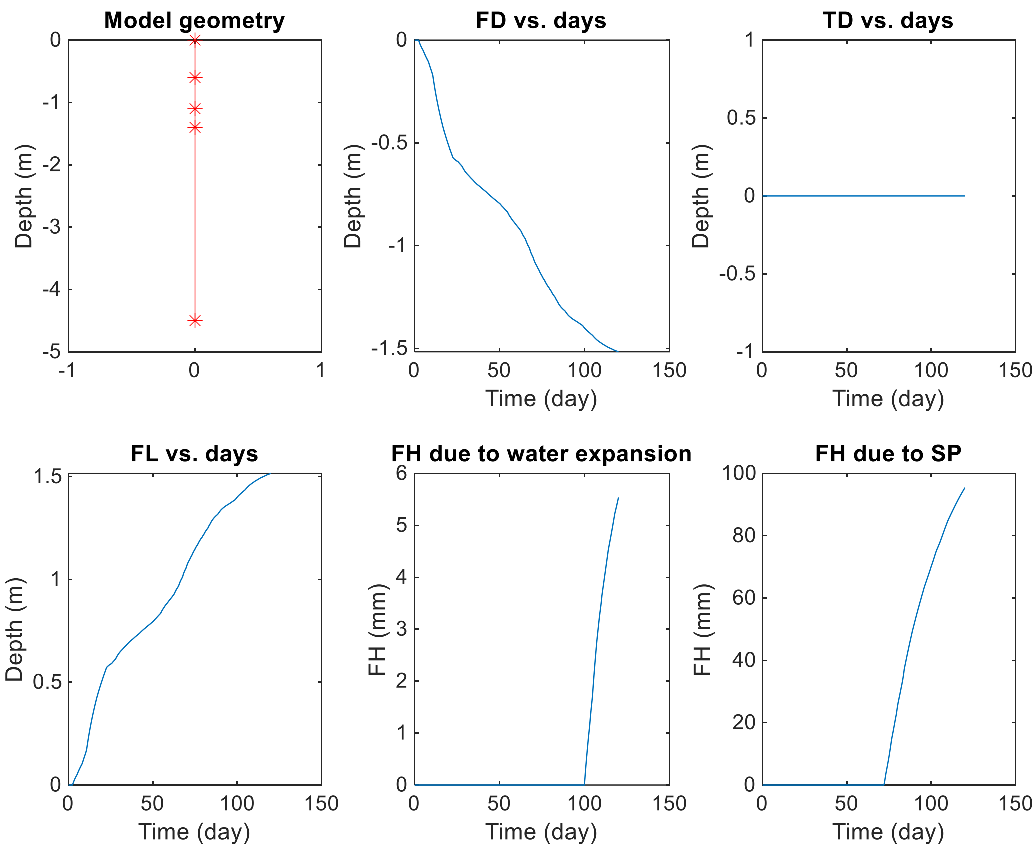


Figure 6-5 Example of MATLAB 1-D model results (used data from Finland site at Joensuu P33)

6.5 Comparison with the modified Berggren's model

According to Bianchini and Gonzalez (2012) the modified Berggren's model can be derived from the Fourier's equation which is the governing equation (6-1) of the simplified 1D model. The calculation process and input requirements are similar between the two models. Hence, the modified Berggren's model is compared with the simplified 1-D model as presented in Table 6-6 .

Table 6-6 Comparison between the modified Berggren's model and the simplified 1D Model

Compared terms	Simplified 1-D model	Modified Berggren's Model
Governing equation	Based on Fourier's law	Derived from Fourier's law
Primary climatic inputs	Daily or monthly average temperature (Freezing and thawing index compatible)	Freezing and thawing index
Ground water influence	Considered	Not considered
Moisture movement	Partially considered due to the consideration of GWT	Not considered (Assumed constant water content)
Coupling consideration	Weak coupling: Precipitation and GWT influence can be incorporated	No coupling
Thermal property inputs	Self-defined (level 1 design) Correlation equations (level 2 and 3 design)	Correlation charts or equations
Initial temperature	Required inputs	No request
Frost heave prediction	Incorporated based on water expansion and segregation theory	Not included

Time series temperature profile	Can compute	Cannot compute
Solving techniques	FEM, Finite difference, MATLAB, etc. (Need to solve PDE)	Hand calculation, Excel, MATLAB or other coding language (No need to solve PDE)

Overall, as shown Table 6-6, the simplified 1D model takes more environmental factors into account for the predicting FD. More incorporated factors indicate more inputs are required for the 1D model. For level 2 and 3, the two models use same correlation equations to compute layer heat conductivity but different ways to compute thermal conductivity. The modified Berggren's model uses a correlation equation by Kersten (1949) to evaluate thermal conductivity; however, Farouki (1981) suggests that Kersten's equations were proposed based on limited soil test results and should be more applicable for soil up to a degree of saturation of 90%, while the thermal conductivity evaluation method by Johansen (1975) generally showed good agreement up to full soil saturation. Hence, method of Johansen (1975) was taken for the 1-D model. About the result, the 1D model can predict time-series frost heave based on segregation theory and water volume expansion. Also, the 1-D model can compute the time series temperature profile along depth. These cannot be achieved by modified Berggren's model. Since solving the 1-D model is essentially solving a PDE with high nonlinearity, performing 1-D model analysis needs more computation power.

6.6 References

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