

**PROPOSED ENHANCEMENTS TO PAVEMENT ME DESIGN: IMPROVED CONSIDERATION OF THE INFLUENCE OF
SUBGRADE SOILS SUSCEPTIBLE TO SHRINK/SWELL AND/OR FROST HEAVE ON PAVEMENT PERFORMANCE**

APPENDIX 4

DETERMINISTIC SHRINK-SWELL MODEL

MAY 2023

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4. SHRINK SWELL DETERMINISTIC MODEL

This document introduces the proposed framework for deterministically estimating the 1D shrink-swell volume change of soil exposed to time-varying climatic effects.

4.1. Summary

This chapter presents an improved framework for estimating the volume change of shrink-swell soils due to time-varying climatic effects using the Lytton et al. (2005) approach with the suction envelope models created by Vann and Houston (2021). The proposed framework for estimating soil volume change of shrink-swell soils as a function of time is presented with an example calculation with data from an AASHTO Long-Term Pavement Performance (LTPP) Seasonal Monitoring Program (SMP) section TX 48-1068 (FHWA, 1995), which is located approximately 80 miles northeast of Dallas, Texas. The framework presented is applicable to uncovered sites where the groundwater table effects are negligible, but it has been calibrated to account for covered areas and for the spatial variation between the pavement center and edges.

4.2. Objectives

The following objectives pertaining to the development of the deterministic shrink swell volume change model were accomplished:

- The following objectives were accomplished as part of this study:
- Review and choose the deterministic SSVC framework which is to be incorporated into the stochastic analysis.
- Development of a framework for stochastically estimating the volume change on shrink-swell soils using the previously developed models for random soil property generation and monthly TMI forecasting.
- Exploration of the stability and sensitivity of the proposed shrink-swell forecast model.
- A comparison of the proposed models to the existing engineering practice including the differences in the uncertainty and sensitivity of the estimates.
- Exploration of the potential implementation of the proposed stochastic shrink-swell soil volume change model to foundation and pavement performance analysis/design.

4.3. Deterministic Shrink-Swell Volume Change (SSVC)

The ability to estimate soil volume change as a function of time is a valuable tool in the design of shallow foundations of pavement structures. Specifically pertaining to pavement design, estimating soil volume change as a function of time allows for the prediction of the potential cumulative International Roughness Index (IRI). The time-varying volume change can also be a valuable tool in the forensic analysis of existing foundation movement of a lightly loaded structure on shallow footings.

The author and members of the ASU research team (Zapata and Mosawi) published a paper in 2021 in the Soil and Rocks International Journal of Geotechnical and Geoenvironmental Engineering titled “An Improved Framework for Volume Change of Shrink/Swell Soils Subjected to Time-Varying Climatic Effects”. The paper presents an improved framework for estimating the volume change of shrink-swell soils due to time-varying climatic effects using the Lytton et al. (2005) approach with the suction envelope models created by Vann and Houston (2021). The proposed framework for estimating soil volume change of shrink-swell soils as a function of time is presented with an example calculation with data from an AASHTO Long-Term Pavement Performance (LTPP) Seasonal Monitoring Program (SMP) section TX 48-1068 (FHWA, 1995), which is located approximately 80 miles northeast of Dallas, Texas. The framework presented is applicable to uncovered sites where the groundwater table effects are negligible, but it has been calibrated to account for covered areas and for the spatial variation between the pavement center and edges.

4.3.1. Suction-Volume Change Relationships

Direct laboratory measurements of the volume change potential of a soil help improve the estimation of potential volume change in the field. The 1-D oedometer “Response to Wetting Test” as described in ASTM D4546 is the common type of laboratory test for volume change determination. One key difference from the laboratory oedometer test compared to the field conditions the soil will experience is the final degree of saturation. The response to wetting test inundates the sample, driving to almost full saturation. However, it is the probability that the soil will reach this moisture level over the period of the structure/pavements design life is very low (Houston and Houston 2017).

A common method for volume change estimation is the Potential Vertical Rise published by the Texas Department of Transportation (TxDOT-12-E, 1978), which includes both empirical-based relationships and result from an oedometer test. In 2005, the Texas DOT updated the approach to determining the volume change of expansive soils using the work by Lytton et al. (2005), which encompassed a suction-based approach. The study concluded that the previous empirical-based approach significantly overestimated the soil heave and did not account for the shrinkage of the soil during dry climatic periods.

A thorough literature review of volume change estimates of unsaturated soil (odometer-based, or suction-based) was performed by Vann (2019). The authors of this paper have carefully reviewed this relative literature summary as part of the research leading up to this paper.

The evaluation of moisture-driven volume change of unsaturated clay soil requires consideration of the net normal (p) and the matric suction (s) stress states. For clays under relatively light confinement, increases in s during drying will typically cause clays to decrease in volume (compress or shrink) and reductions in s during wetting will cause increases in volume (expansion or swell). Clay soils at high net normal stress states can also reduce in volume (collapse) during wetting (Houston and Zhang, 2021; Nooray, 2017). The volumetric response is commonly expressed in terms of changes in void ratio (e), and the general relationship between e , p and s for clay soils is represented using three-dimensional (3D) state surfaces (Alonso et al., 1994, 1999; Delage & Graham, 1996; Fredlund & Morgenstern, 1976; Gens & Alonso, 1993; Gens et al., 2016; Wheeler & Sivakumar, 1995; Vu & Fredlund, 2004; Wray et al., 2005; Zhang & Lytton, 2009a, 2009b). For many foundation and pavement applications in engineering practice, vertical deformations govern the design which allow for the e - p - s relationship to be analyzed using one-dimensional (1D) at-rest (K_0) conditions - or at least the 1-D analyses provide adequate information for decision making (Adem & Vanapalli, 2013; Fredlund et al., 1980; Houston & Houston, 2018; Lytton, 1997; Nelson & Miller, 1992; Nelson et al., 2015; Overton et al., 2006).

The study presents a comparison of the matric suction-volume change indices (for 1-D monotonic loading by wetting or drying) of intact clays (natural soil stress states) using relatively undisturbed specimens tested by Olaiz (2017), to those of reconstituted/compacted specimens measured by Singhal (2010) and Mosawi (2022) using OPPDs, including the Fredlund SWCC (SWC-150) developed by GCTS in Tempe, AZ. Singhal (2010) created a unique database from laboratory measurement of the mechanical response of clay soils under imposed conditions of both stress state variables using the OPPD, with attention on evaluation of a substantial portion of the void ratio state surface for clay soils. Olaiz (2017) and Mosawi (2022) continued the exploration of testing relatively intact and reconstituted soil specimens in the OPPD, respectively. The materials tested by Olaiz and Mosawi were obtained from locations associated with expansive soil areas in San Antonio, TX, and Denver, CO (same study sites previously analyzed in this document). The compilation of data from these extensive laboratory efforts was used to provide qualitative and quantitative comparisons of the mechanical response of reconstituted to relatively intact specimens.

The comparison of the matric suction-volume change indices of the intact samples from Olaiz (2017) resulted in significantly lower magnitude volume change than those of the remolded samples tested by Singhal (2010) and Mosawi (2022) for similar range of wPI specimens and suction range. A brief statistical analysis using descriptive

statistics, box plots, and 1:1 comparison indicated that for the compiled dataset, wPI provides a useful indicator for volumetric response for remolded specimens under relatively light confining stress. The data also demonstrated the critical role of stress history in shrink/swell response of clays to changes in moisture state. The data from the two-stress state controlled OPPD results provides a substantial start for development of $e-p-s$ relationships and correlations for expansive clay soils.

4.4. Framework Outline

The following outline summarizes the steps of the improved framework for estimating the volume change of shrink/swell soils due to time-varying (monthly) climatic effects:

1. Weather station identification and data extraction
2. 30-year and monthly Thornthwaite Moisture Index per Witczak et al. (2006)
3. Determination of equilibrium suction envelope parameters per Vann and Houston (2021): depth to equilibrium suction and magnitude of equilibrium suction
4. Back-calculation of variables for Mitchell's (1979) equation
5. Development of long-term wet and dry suction profiles
6. Initial estimation of monthly changes in suction at the surface per Perera (2003)
7. Fourier equation fit to the monthly suction change at the surface.
8. Generation of monthly suction profile
9. Suction profile adjustments for varying surface boundary conditions
10. Calculation of net normal stress profile
11. Estimation of suction compression index (assuming value is not directly measured)
12. Calculation of strain monthly
13. Calculation of volume change monthly

The above deterministic framework is used in the stochastic analysis presented herein. Figure 4-1 presents a flow chart for the deterministic SSVC procedure.

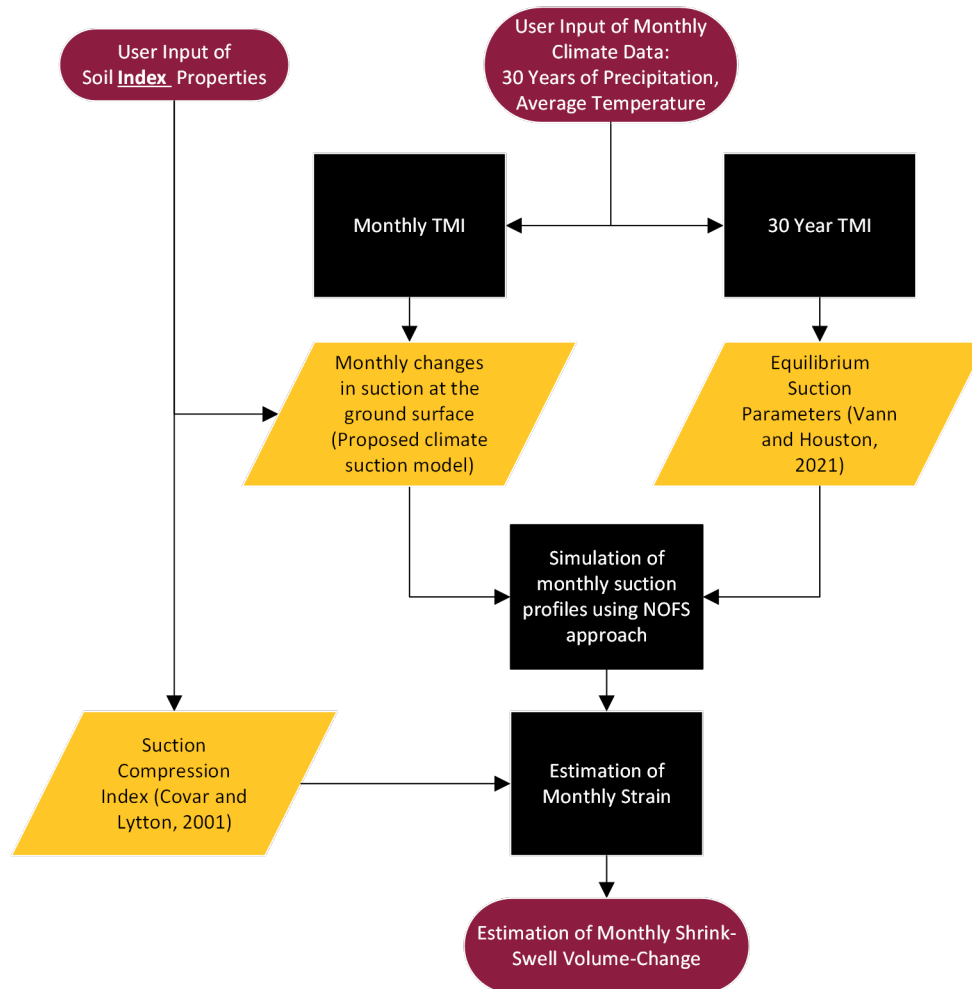


Figure 4-1 Flow of the Deterministic SSVC Analysis Procedure

Step 1: Climate Data

An SMP pavement section approximately 80 miles northwest of Dallas, Texas (TX 48-1068) is used to provide an example for the proposed framework. For the purposes of this example calculation, the climate data was gathered from the weather station nearest to the site and identified using the open-access Thornthwaite Moisture Index (TMI) GIS map developed by Olaiz et al. (2017), which uses the National Oceanic and Atmospheric Administration's (NOAA) 30-year climate normal database for the United States. Figure 4-2 below presents an excerpt for the GIS map, which has the Paris, TX weather station selected.

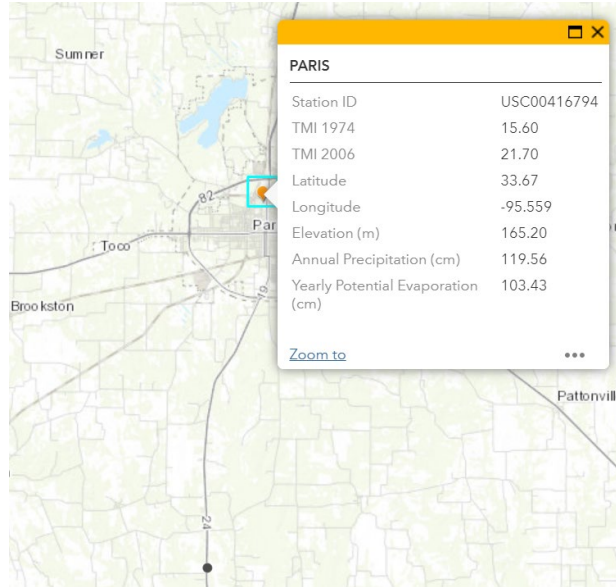


Figure 4-2: Paris, TX weather station (NOAA ID USC00416794) data from online TMI GIS map (Olaiz et al., 2017)

The “Station ID” shown in the pop-up window in **Figure 4-2** (USC00416794) is the only data needed for the purposes of this study. However, the remaining data shown may be helpful to get an understanding of the general climatic conditions at the site.

The NOAA climate data associated with each station in the country can be extracted from the online NOAA FTP site. It is recommended that the extracted weather data be filtered to contain the necessary variables for computation of the Thornthwaite Moisture Index (Witczak et al., 2006):

- Year
- Month
- Monthly Precipitation (cm)
- Monthly Average Temperature (Celsius)

Note that the Vann and Houston (2021) models used in the proposed analysis correlate the suction envelope parameters to a 30-year TMI value. As such, the climate data from the NOAA database for station USC00416794 was extracted for the date range of 9/1967 to 9/1997 (the last date of measured data from the SMP study for the TX 48-1068 section).

Step 2: Monthly and 30-year Thornthwaite Moisture Index (Witczak et al., 2006)

To determine yearly TMI for each month, first the potential evapotranspiration (*PET*) for each month must be calculated.

$$PET(cm) = f_1 f_2 1.6 \left(\frac{10t}{I} \right)^a \quad (4-1)$$

where,

f_1 is the fraction of the number of days in month divided by the average number of days in month, 30; f_2 is the fraction

of the number of hours in a day divided by the base of 12 h in a day; t is the mean monthly temperature in degrees Celsius; I is the annual heat index; and a is a coefficient.

$$I = \sum_{i=1}^{12} \left(\frac{t_i}{5} \right)^{1.514} \quad (4-2)$$

Where,

t_i is the mean temperature for the i^{th} month, and

$$a = 6.75I^3 \times 10^{-7} - 7.71I^3 \times 10^{-5} - 1.792I \times 10^{-2} + 0.49239 \quad (4-3)$$

The TMI (Witczak et al., 2006) can now be determined by

$$TMI = 75 \left(\frac{P}{PET} - 1 \right) + 10 \quad (4-4)$$

where,

P is the precipitation for the given month.

To visualize the climate data over time, the monthly average temperature, monthly rainfall, and the calculated TMI can be plotted (Figure 4-3). For the example calculation at the TX 48-1068 SMP section, the 30-year weather data must be analyzed (9/1967 to 9/1997). The duration which measured elevation change data was collected by SMP (3/1987 to 9/1997) is also analyzed for the example calculation so that measured vs. predicted values can be compared.

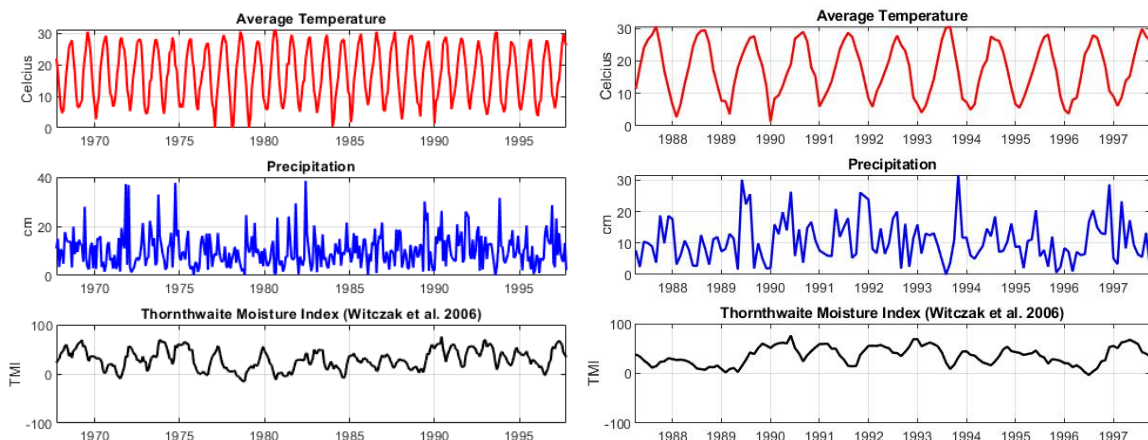


Figure 4-3: Monthly average temperature and rainfall data for NOAA weather station USC00416794 with the calculated yearly TMI (Witczak et al, 2006) between a.) 9/1967 and 9/1997 and b.) 3/1987 and 9/1997.

The 30-year TMI value (Witczak et al., 2006) calculated from the NOAA data set for the USC00416794 station is +29.6. This value differs slightly from the +21.7 value previously shown on the TMI GIS map (**Figure 2**) due to the difference in date ranges used in the Olaiz et al. (2017) study.

Step 3: Suction Envelope Parameters (Vann and Houston, 2021)

The suction envelope defines the maximum and minimum suction values at the surface and within the subsurface to a depth of stable suction. The suction envelopes are established using the following parameters: (1) equilibrium suction, ψ_e , (2) depth to equilibrium suction, D_{ψ_e} , (3) change in suction at the ground surface, $\Delta\psi$, and (4) climate parameter, r .

The depth at which the climate-driven fluctuations in soil suction begins to stabilize, or equilibrate is determined using the Vann and Houston (2021) model (Figure 4-4) relatively flat uncovered sites subjected to natural climate surface flux conditions:

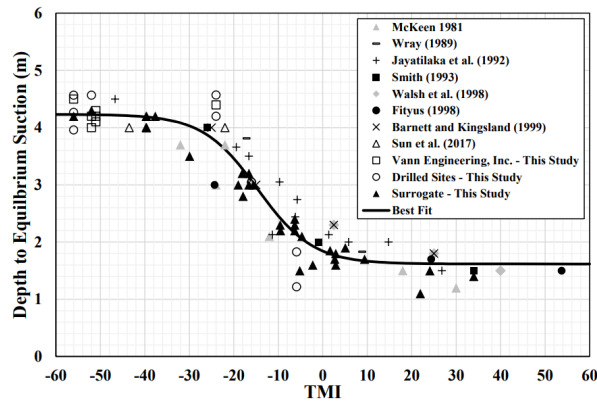


Figure 4-4: Relationship between TMI and the depth to constant soil suction for uncovered and non-irrigated sites (Vann and Houston, 2021)

The equation for the Depth to Equilibrium Suction (D_{ψ_e}) versus TMI regression shown above is:

$$D_{\psi_e} = 1.617 + \frac{2.617}{1 + e^{(2.36 + 0.1612TMI)}} \quad (4-5)$$

With an $R^2 = 0.9045$ and standard error = 0.3147 m.

The stable, or equilibrium, suction value is determined using the Vann and Houston (2021) model (Figure 4-5). The soil suction unit of pF (log to the base 10 of soil suction in centimeters of water) was used in the Vann and Houston (2021) study due to its extensive use in the geotechnical practice, with regards to unsaturated soils. Note that log of suction in kPa units is approximately equal to suction in pF units, minus 1 (i.e., 4.0 pF = 3.0 log (suction (kPa))).

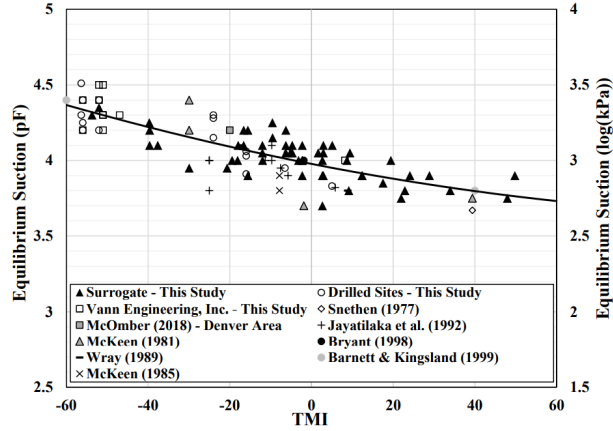


Figure 4-5: Equilibrium Suction vs. TMI with Literature Values (Vann and Houston, 2021)

The equation for the Equilibrium Suction (ψ_e) as a function of TMI is:

$$\psi_e(pF) = 0.0002TMI^2 - 0.0053TMI + 3.9771 \quad (4-6)$$

With an $R^2 = 0.6539$ and a standard error = 0.1959 pF.

The limits of the potential surface flux, or potential change in suction at the surface, is determined using the Vann and Houston (2021) model (Figure 4-6)

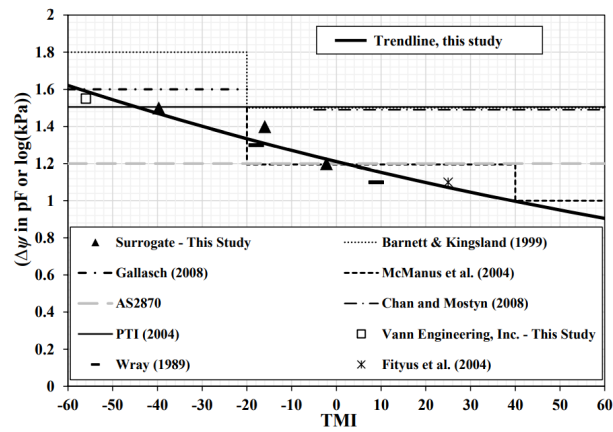


Figure 4-6: Limits of the potential change in suction at the surface vs. TMI with literature values (Vann and Houston, 2021)

The equation for the potential change in suction at the surface ($\Delta\psi$) as a function of TMI, as shown above is:

$$\Delta\psi(pF) = 1.2109e^{(-0.005TMI)} \quad (4-7)$$

With an $R^2 = 0.9184$ and a standard error = 0.1835 pF.

Aubeny and Long (2007) presented illustrative suction envelopes, developed from unsaturated flow analyses (Mitchell, 1980), to demonstrate that asymmetrical soil suction envelopes are expected, depending on the climate (TMI). Aubeny and Long introduced a climate parameter, r , that is the percentage of the total anticipated change in soil suction at the surface, $(\Delta\psi)$, comprising the wet side of the suction envelope. The climatic parameter can be expressed in terms of the equilibrium suction and the minimum (wet) and maximum (dry) suction at the surface ($z=0$):

$$r = \frac{(\psi_e - \psi_{wet_{z=0}})}{(\psi_{dry_{z=0}} - \psi_{wet_{z=0}})} = \frac{(\psi_e - \psi_{wet_{z=0}})}{\Delta\psi} \quad (4-8)$$

Houston and Vann created a relationship between the climatic parameter and TMI (Figure 4-7).

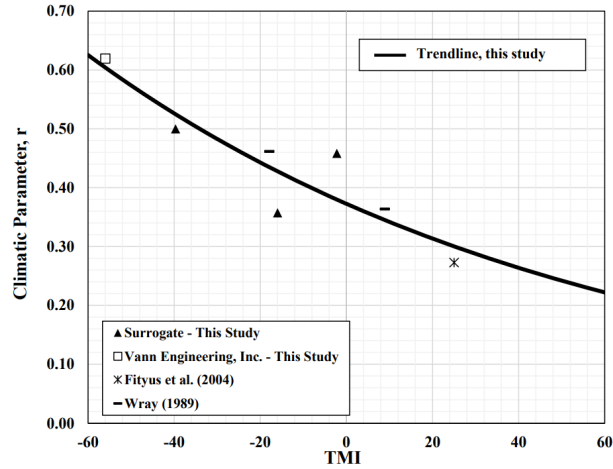


Figure 4-7: Relationship between the Climate Parameter, r , and TMI (Vann and Houston, 2021)

The equation for the climatic parameter (r) as a function of TMI as shown above is:

$$r = 0.3725e^{(-0.009TMI)} \quad (4-9)$$

With an $R^2 = 0.7998$ and a standard error = 0.1132.

Table 4-1 presents the suction envelope parameters for the SMP TX 48-1068 section which had 30-year TMI of 29.6.

Table 4-1: Suction envelope parameters for the SMP TX 48-1068 per Vann and Houston (2021) with a TMI = 29.6

Suction Envelope Parameter	Value
Depth to Equilibrium Suction (D_{ψ_e})	1.62 m
Equilibrium Suction (ψ_e)	3.84 pF
Change in suction at the surface ($\Delta\psi$)	1.044 pF
Climatic parameter (r)	0.2854

Step 4: Back Calculation of variables for Mitchell's equation (1980)

The suction envelope can now be generated using the simplified unsaturated flow equation derived by Mitchell in 1980. The adjustment to the equation by Aubeny and Long (2007) for asymmetrical suction envelopes has also been incorporated into this study.

The Mitchell (1979) equation for change in suction with depth and time, simplified by Naiser (1997) to consider only the extreme suction cases (wet and dry), by taking the time variable to infinity, is used to obtain the shape of the envelopes.

$$\psi(z) = \psi_e + \Delta\psi e^{\left(-z\sqrt{\frac{n\pi}{\alpha}}\right)} \quad (4-10)$$

where,

ψ is units of pF and z , n and α are in consistent units, $\psi(z)$ is the suction value at any depth z , n is the frequency of suction cycles, and α is the diffusion coefficient.

The suction change with depth is a function of change in suction at the surface ($\Delta\psi$ in pF units) and the equilibrium suction (ψ_e).

Once the key parameters of the profiles are established for any given TMI, all information required for the Mitchell (1980) flow computations (such as the ratio of the diffusion coefficient to the number of seasonal cycles per year) can be back-calculated (Vann, 2019). The n , α , π terms in the Mitchell (1981) equation, are back-calculated using the known equilibrium depth, D_{ψ_e} , change in suction at surface, $\Delta\psi$, and the 0.2 pF difference (Lytton, Aubeny, and Long, 2005; Vann 2019), wet to dry, at the depth of equilibrium.

$$\frac{n\pi}{\alpha} = \left(\frac{\ln\left(\frac{0.2 pF}{\Delta\psi}\right)}{-D_{\psi_e}} \right)^2 \quad (4-11)$$

The suction profile can now be generated using equations 10 and 11 and the previously computed components of the surrogate suction, where suction is in pF units and depth is in meters.

Step 5: Development of the wet and dry suction envelope

The suction envelope defines the boundary conditions for the suction value at any depth within the soil profile. At the ground surface, the minimum (wet) and maximum (dry) suction values can be determined using the following expressions:

$$\psi_{wet_{z=0}} = \psi_e - r\Delta\psi \quad (4-12)$$

$$\psi_{dry_{z=0}} = \psi_{wet_{z=0}} + \Delta\psi \quad (4-13)$$

The minimum (wet) and maximum (dry) suctions for the TX 48-1068 section are 3.54 and 4.58, respectively.

The step size, or thickness of depth intervals (dz) must be determined. A sensitivity analysis should be performed to determine the number of steps (n_z) needed for the analysis; however, a value of 20 steps has been found to be sufficient for the volume change calculation performed in this study. The step size is computed by:

$$dz = \frac{D_{\psi_e}}{(n_z - 1)} \quad (4-14)$$

The step size for the SMP TX 48-1068 section is 8.526 cm using 20 steps with a depth of equilibrium suction of 1.62 m.

The wet and dry limit suction curves are iteratively calculated as the depth (z) is increased from 0 (ground surface) to the depth of equilibrium suction.

$$\psi(z_i)_{wet} = \psi_e - r\Delta\psi e^{\left(-z_i\sqrt{\frac{n\pi}{\alpha}}\right)} \quad (4-15)$$

$$\psi(z_i)_{dry} = \psi_e + (1-r)\Delta\psi e^{\left(-z_i\sqrt{\frac{n\pi}{\alpha}}\right)} \quad (4-16)$$

The suction for the SMP TX 48-1068 section is shown in Figure 4-8.

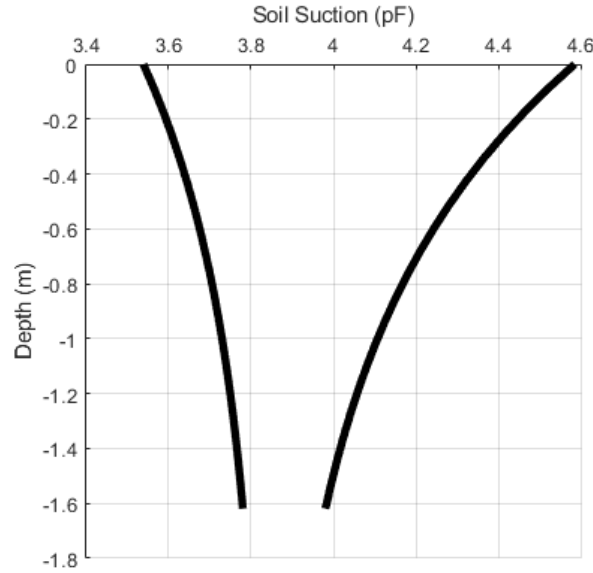


Figure 4-8: Suction envelope for the SMP TX 48-1068 section using the Vann and Houston (2021) models

Step 6: Initial estimate of monthly changes in suction at the surface (Perera, 2003)

It is important to note that the following steps for determining the suction at the surface over time are specific to a deterministic approach for estimating historic ground movements. Such approach can be used for a case study, forensic analysis, or calibration efforts based on measured data. The suction at the surface over time can also be modeled using a stochastic analysis with randomly generated monthly TMI values based on the historic averages and standard deviations. The second type of analysis can be used for designs of future structures; however, an example of such analysis is not presented in this paper.

In 2003, Perera studied the relationship between in-situ moisture content, suction, TMI, and index soil properties. He developed correlations for two models: the *TMI-P₂₀₀* model, which is valid for granular base materials; and the *TMI-P₂₀₀/wPI* model, used to estimate the equilibrium suction of subbase and subgrade materials (Rosenbalm 2011). The two models are briefly explained in the following paragraphs.

The *TMI-P₂₀₀/wPI* model is of interest to this study. This model was developed for fine-grained material, which makes it suitable for expansive soils. For such materials, in addition to P_{200} , the weighted plasticity index, wPI , property was added, where:

$$wPI = \left(\frac{P_{200}}{100} \right) PI \quad (4-17)$$

The wPI for the example in Paris, TX site is 18.4 based on the percent passing the #200 sieve of 74% and a PI of 20.

The following equation is used to calculate suction based TMI , P_{200} , and wPI (Perera 2003).

$$\Psi = \alpha \left[e^{\left[\frac{\beta}{TMI + \gamma} \right]} + \delta \right] \quad (4-18)$$

where, Ψ is the matric suction of the soil; and α, β, γ , and δ are regression constants.

Rosenbalm developed equations for each regression constant. These equations are used when wPI is less than 0.5 (Rosenbalm 2011):

$$\beta = 2.56075(P_{200}) + 393.4625 \quad (4-19)$$

$$\gamma = 0.09625(P_{200}) + 132.4875 \quad (4-20)$$

$$\delta = 0.025(P_{200}) + 14.75 \quad (4-21)$$

The following equations are used when $wPI \geq 0.5$:

$$\beta = 0.006236(wPI)^3 - 0.7798334(wPI)^2 + 36.786486(wPI) + 501.9512 \quad (4-22)$$

$$\gamma = 0.00395(wPI)^3 - 0.04042(wPI)^2 + 1.454066(wPI) + 136.4775 \quad (4-23)$$

$$\delta = -0.01988(wPI)^2 + 1.27358(wPI) + 13.91244 \quad (4-24)$$

Step 7: Adjustment to the estimation of monthly changes in suction at the surface (Vann and Houston (2021))

It has been observed by the authors that the suction at the surface calculated using the Perera (2003) model typically will not reach the long-term minimum and maximum suction values observed by Vann and Houston (2021). This may not cause a significant issue if the analysis period is relatively short (e.g., less than ten years), however; for the purpose of pavement design, which typically incorporates a design life of 20+ years, it is recommended that the surface suction values determined from the Perera model be adjusted so that they will reach the limits observed by Vann and Houston (2021). This can be conducted by normalizing the maximum and minimum suction values from the Perera model to the previously computed potential change in suction at the surface ($\Delta\psi$).

$$(\psi_i)_{norm} = (\psi_{wet})_{z=0} + \Delta\psi \left(\frac{(\psi_i)_{z=0} - (\psi_{Perra})_{min}}{(\psi_{Perra})_{max} - (\psi_{Perra})_{min}} \right) \quad (4-25)$$

After iterating the process for each month, the adjusted surface suction values can be plotted to help visualize

adjustment (Figure 4-9).

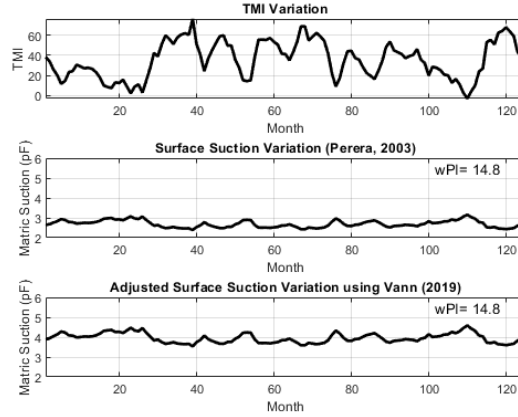


Figure 4-9: Monthly TMI, Perera (2003) surface suction, and the Vann (2019) adjusted surface suction for the TX 48-1068 section for the date range 3/1987 to 9/1997

Step 8: Fourier equation fit to the monthly surface suction (Aubeny and Long, 2007)

In order to model the suction changes as a function of time and depth, an equation must be developed to represent the variation of suction at the surface. Typically, a simple sinusoidal fit has been used to represent the surface suction variation with time. However, Aubeny and Long (2007), proposed that a Fourier transform can be used to improve the goodness of fit. As such, an 8th degree Fourier series is used in this analysis to fit an equation to the highly variable surface suction data.

In general, the Fourier series is a sum of sine and cosine functions that describes a periodic signal. It is represented in either the trigonometric form or the exponential form.

$$y = a_0 + \sum_{i=1}^n a_i \cos(iwx) + b_i \sin(iwx) \quad (4-26)$$

Where, x represents time (for this analysis), a_0 models a constant (intercept) term in the data and is associated with the $i = 0$ cosine term, w is the fundamental frequency of the signal, n is the number of terms (harmonics) in the series, and $1 \leq n \leq 8$, and a_i and b_i are the fitting parameters. Additional background information on the Fourier series can be found at: (<https://www.mathworks.com/help/curvefit/fourier.html>).

Figure 4-10 presents the 1st and 8th order Fourier fit to the Vann and Houston (2021) adjusted surface suction for the TX 48-1068. The 1st order fit closely represents the original approach to modeling the surface suction flux by Lytton et al. (2005) using Mitchell's (1979) equation. The adjusted R^2 for the 1st order Fourier fit to the suction data is 0.2903, while the 8th order Fourier fit increases the adjusted R^2 to 0.7056.

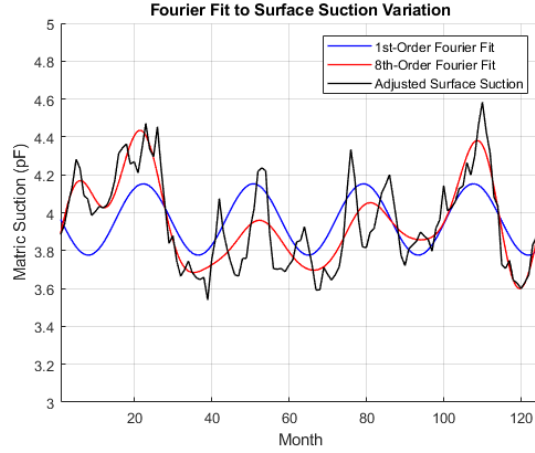


Figure 4-10: 1st and 8th order Fourier fit to the surface suction data for the TX 48-1068 section for the date range 3/1987 to 9/1997

One limitation of requiring an equation to represent the surface suction, is that generally the equation fit will not be able to encompass the maximum and minimum values of the individual monthly data. For purposes of the shrink/swell volume change analysis, the inclusion of the peaks of the surface suction can provide more accurate and conservative representation of extreme events. As such, the Fourier surface suction equation can be normalized between the maximum and minimum values of the surface suction; however, this additional step was not performed as part of the example analysis presented in this paper.

Note that the initial suction is a function of the TMI value for that month. The initial suction (time = 0), can be adjusted using a phase shift of the Fourier equation. Lytton et al. (2005), provided values of phase shifts for different initial conditions of the soil (wet, dry, and equilibrium). The example calculation in this report does not include the phase shift for the initial conditions.

Step 9: Monthly Suction Profile (Aubeny and Long, 2007)

The next step is to model the suction profile change with time using Aubeny and Long's (2007) adjusted 1979 Mitchell equation.

$$u(y, t) = U_e - (U_{dry} - U_{wet}) \sum_{k=1}^{\infty} \alpha_k \exp(-\sqrt{\lambda k}) \cos(\tau k - \sqrt{\lambda k}) \quad (4-27)$$

Where

$$y = \text{depth}, \lambda = \pi y^2 n_\lambda / \alpha, \tau = 2\pi t n_\lambda, \text{ and } \alpha_k = (2 / k\pi) \sin(k\pi r) \quad k = 1, 2, 3, \dots$$

For a given point in time, the suction profile can be estimated using the Aubeny and Long approach. Figure 4-11 presents the estimated suction at time = 1 month for the TX 48-1068 SMP site. Note that the long-term or extreme boundaries of the suction profile are known from the Vann (2019) correlations with the 30-year TMI value previously presented herein.

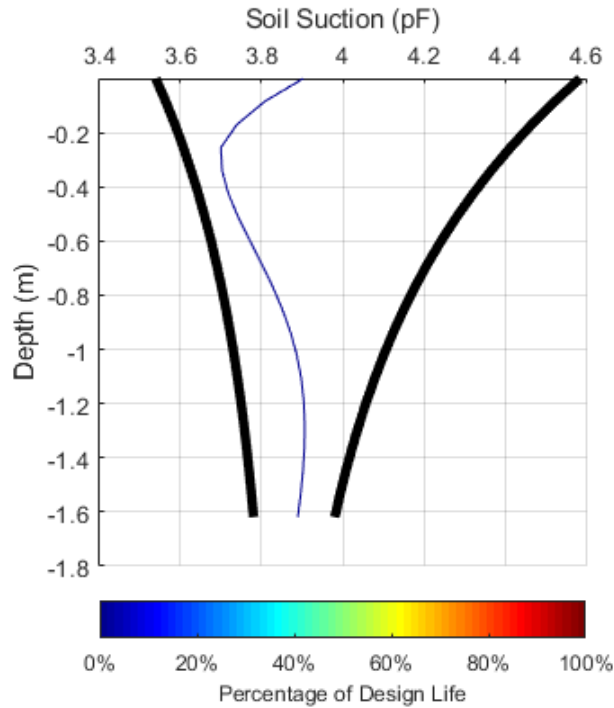


Figure 4-11: The estimated initial suction profile (time = 1 month) for TX 48-1068 section

The monthly change in suction at each depth can be determined by calculating the following months suction profile and taking the difference of the two values at each depth. It is this ongoing change in soil suction with time that drives the volume change of shrink/swell soils. Figure 4-12 shows the suction profiles for the initial state ($t = 1$ month) and the second month ($t = 2$ months) for the TX 48-1068 SMP site. Figure 4-13 shows the suction profiles for month 1 through month 12.

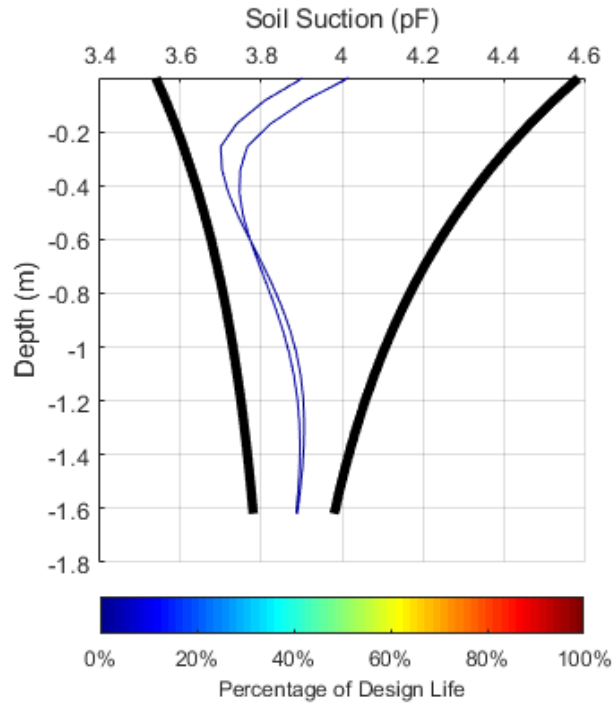


Figure 4-12: The estimated initial suction profiles (time = 2 months) for TX 48-1068 section

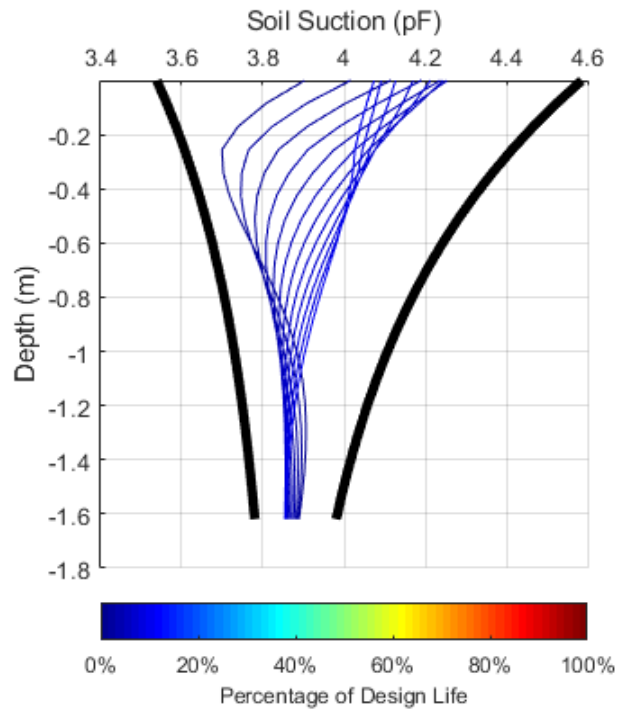


Figure 4-13: The estimated initial suction profiles (time = 12 months) for TX 48-1068 SMP section

Figure 14 presents the monthly suction profiles over the date range 3/1987 to 9/1997 life for TX 48-1068 SMP section.

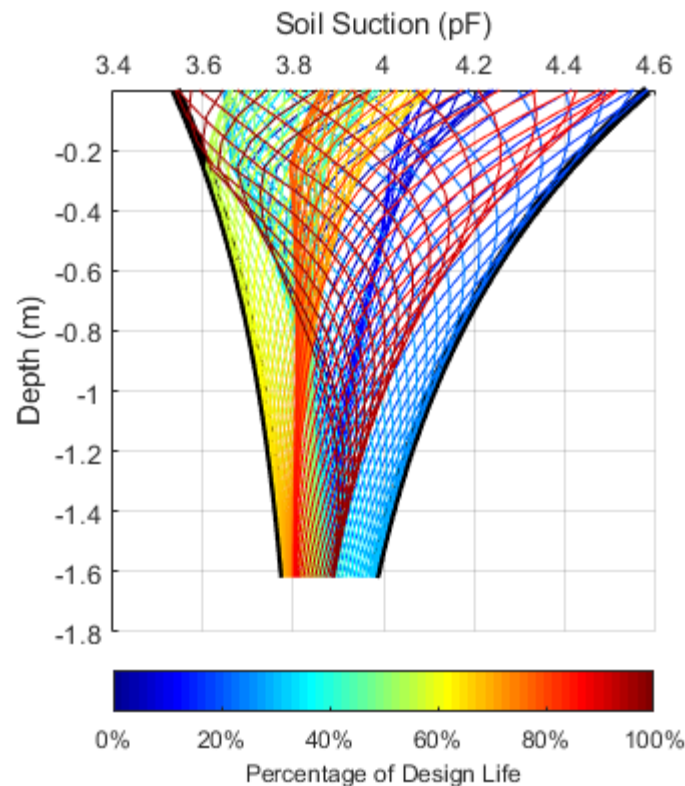


Figure 4-14: Monthly suction profiles over the date range 3/1987 to 9/1997.

From the figure, the significant swings of the suction profile from wet to dry, over the date range 3/1987 to 9/1997, can be observed. The monthly change in suction at each depth in the soil profile can be determined from this model.

To account for hysteresis effects associated with the wetting and drying of soil, it is important to record if the soil is wetting or drying at each depth and time during the iterative analysis. This information will be used during the strain calculation discussed later in this report.

Step 10: Suction Profile Adjustments due Varying Boundary Conditions

It is plausible that the subject site has a variable groundwater table, nearby vegetation, or is constructed using a moisture barrier. If any of these boundary conditions are present, the suction envelope and profile must be adjusted. (e.g. the suction profile will reach osmotic suction at the groundwater table depth). No variable boundary conditions were included in this example calculation.

Step 11: Net Normal Stress Profile

The net normal, or overburden stress, is a key component of shrink/swell volume change determination as it will help reduce potential soil swelling and can increase soil shrinkage. The overburden stress profile is determined using the conventional total stress approach.

$$\sigma_z = \sum (\gamma_{soil} z) \quad (4-28)$$

Note that the water content is subject to change over time as the soil suction changes, which will affect the magnitude of the net normal stress. However, for purposes of this example calculation, the effect of the changing water content on the net normal stress is negligible and is not included in the analysis.

If there are foundation loads or increases overburden stresses due to pavement layers above the subject soil profile, an increase of stress will be applied throughout the net normal stress profile.

Step 12: Suction Compression Index (Covar and Lytton, 2001)

The most widely accepted method for estimating volumetric strain is the one developed for the Texas DOT and the Federal Highway Administration, FHWA, by Lytton et al. in 2005, which is as follows:

$$\frac{\Delta V}{V} = -\gamma_h \log \left(\frac{h_f}{h_i} \right) - \gamma_\sigma \log \left(\frac{\sigma_f}{\sigma_i} \right) - \gamma_\pi \log \left(\frac{\pi_f}{\pi_i} \right) \quad (4-29)$$

Where, $\frac{\Delta V}{V}$ is the volumetric strain (volume change with respect to initial volume); γ_h is the the matric suction compression index; γ_σ is the mean principal stress compression index; γ_π is the osmotic suction compression index; h_i is the initial matric suction; h_f is the final matric suction; σ_i is the initial mean principal stress; σ_f is the final mean principal stress; π_i is the initial osmotic suction; and π_f is the final osmotic suction.

Although, total suction is the sum of matric suction and osmotic suction, Fredlund wrote “Matric suction in a soil mass change is a result of moisture infiltration and evaporation at the ground surface. Osmotic suction in the soil does not appear to be highly sensitive to modest changes in the water content of the soil. As a result, a change in the total suction is quite representative of a change in the matric suction.” (Fredlund 2012). Also, Lytton wrote: “It is the change of matric suction that generates the heave and shrinkage, while osmotic suction rarely changes appreciably.” (Lytton 2005) Thus, the change in matric suction is responsible to shrinkage and heave and osmotic suction does not affect enough to be concerned. (Lytton 2005, Fredlund 2012) Thus, the equation can be rewritten as:

$$\frac{\Delta V}{V} = -\gamma_h \log \left(\frac{h_f}{h_i} \right) - \gamma_\sigma \log \left(\frac{\sigma_f}{\sigma_i} \right) \quad (4-30)$$

Note that the net normal stress portion of the equation is added if the soil is wetting (swelling) and subtracted if the soil is drying (shrinking).

The Suction compression index, γ_h , is a parameter used to relate total suction to volume change to predict heave or shrinkage in expansive soils. This value can either be measured or estimated using soil index properties (Atterberg limits and gradation) as described in Covar and Lytton (2001).

First, the mineralogical zone is determined using Figure 4-15 with soils plasticity index (PI) and liquid limit (LL).

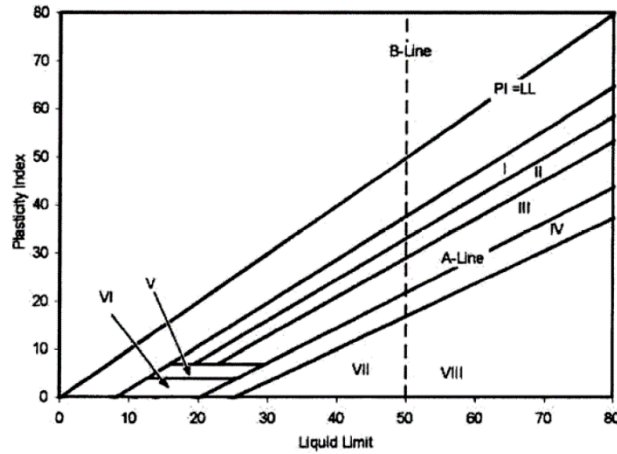


Figure 4-15: Mineralogical zones for soil (Covar and Lytton, 2001)

The zone for the TX 48-1068 SMP section site is Zone 2, using a LL = 60 and a PI = 40.

The percent fine clay (%fc) is then calculated using the percent passing #200 sieve (P_{200}) and the percent clay (%clay) obtained via hydrometer testing.

$$\%fc = \left(\frac{\%clay}{P_{200}} \right) \quad (4-31)$$

The %fc for the example site is 21.05%, using a %clay = 20% and $P_{200} = 95\%$.

The average suction compression index (γ_0) can now be determined using the charts developed by Covar and Lytton (2001), which are separated by mineralogical zones (Figure 4-16)

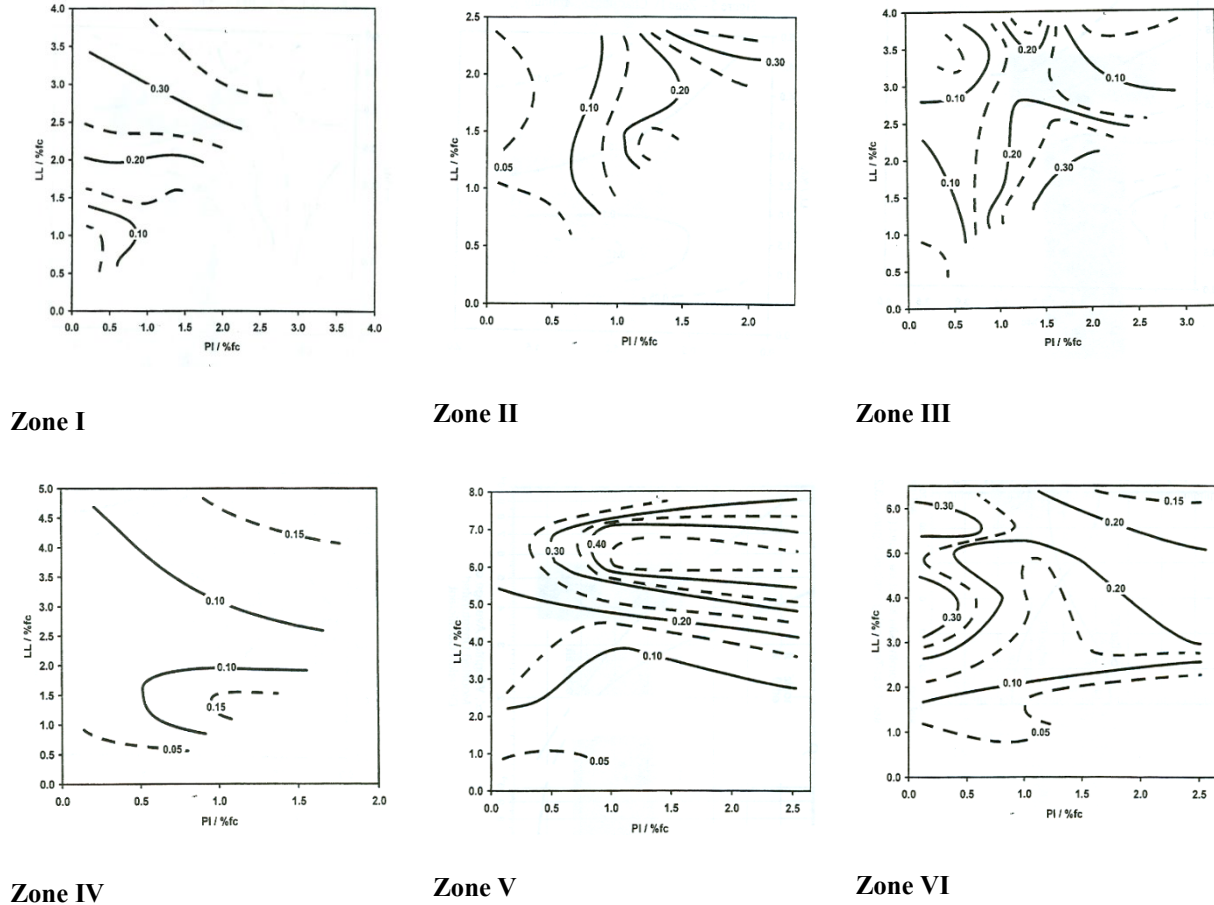


Figure 4-16: Suction Compression Index based on Mineralogical Classification and Soil Index Properties (Covar and Lytton 2001)

The average suction compression index for the TX 48-1098 SMP section is 0.051 using Zone 1, %fc = 43.78%, LL = 38, and PI = 20.

The adjusted suction compression index (γ_h) is now determined by:

$$\gamma_h = \gamma_0 (\%fc) \quad (4-32)$$

The adjusted suction compression index for TX 48-1098 SMP section site is 0.0223.

The hysteresis effects of the soil are now accounted for using the equations from the PTI (2006) Manual. The wetting and drying suction compression indices must be calculated for each depth and time of the analysis using the recorded wetting/drying information from Step 9.

$$\gamma_{swell} = \gamma_h e^{(\gamma_h)} \quad (4-33)$$

$$\gamma_{shrink} = \gamma_h e^{(-\gamma_h)} \quad (4-34)$$

Step 13: Monthly Strain Calculation

The mean principal stress compression index, γ_σ , can be calculated using its relation to the compression index, C_c , and void ratio, e , as follows (Lytton 2005):

$$\gamma_\sigma = \frac{C_c}{1 + e_0} \quad (4-35)$$

Where, C_c is the compression index; and e_0 is the void ratio. For purposes of this example calculation, the mean principal stress compression index was assumed to be 10% of the suction compression index as recommended by Lytton (2005).

The mean principal stress must be iteratively determined at each depth and time step, as it is a function of the net normal stress and the wetting/drying condition.

$$\sigma = \frac{1 + 2K_0}{3} \sigma_z \quad (4-36)$$

Where, σ_z is the previously calculated vertical stress at a point below the surface in the soil mass; and K_0 is the 1-D at-rest lateral earth pressure coefficient.

$$K_0 = e^{\left(\frac{1 - \sin(\phi')}{1 + \sin(\phi')} \right) \left(\frac{1 + d \sin(\phi')}{1 - k \sin(\phi')} \right)^n} \quad (4-37)$$

Values of coefficients e , d , k , and n for different soil conditions are given in Table 4-2.

Table 4-2: Lateral Earth Pressure Parameter Coefficients

Conditions	K_0	e	d	k	n
Cracked	0	0	0	0	1
Drying (Active)	1/3	1	0	0	1
Equilibrium (at rest)	1/2	1	1	0	1
Wetting (within movement active zone)	2/3	1	1	0.5	1
Wetting (below movement active zone)	1	1	1	1	1
Swelling near surface (passive earth pressure)	3	1	1	1	2

The angle of internal friction, ϕ' , can be estimated from its empirical correlation with plasticity index, PI, based on triaxial compression tests.

$$\phi' = 0.0016PI^2 - 0.3021PI + 36.208 \quad (4-38)$$

The internal angle of friction for the TX 48-1068 SMP site is 30.8° using a $PI = 20$.

Using the data developed from the iterative steps discussed above, and the suction-overburden-strain relationships, the volume change over time can be estimated. Figure 4-17 presents the volume change estimation for the Paris, TX site.

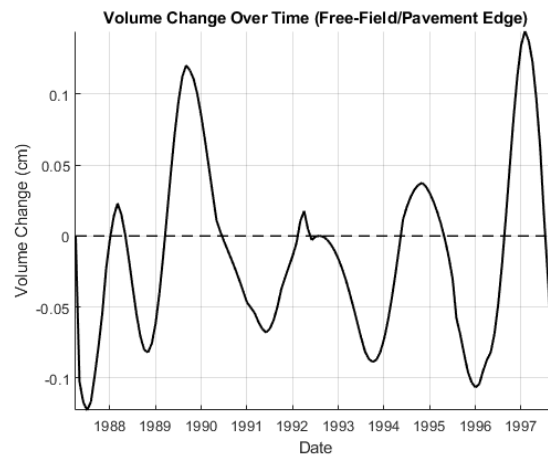


Figure 4-17: Volume change over time for the TX 48-1068 site

4.4.1. Estimated Volume Change Comparison to Measured Data

The estimated volume change from the proposed framework for the TX 48-1068 site was compared to the measured data gathered from the LTTP SMP database. The estimated data was normalized to the value of the first measurement. Figure 4-18 presents the measured and estimated volume change for the TX 48-1068 SMP section.

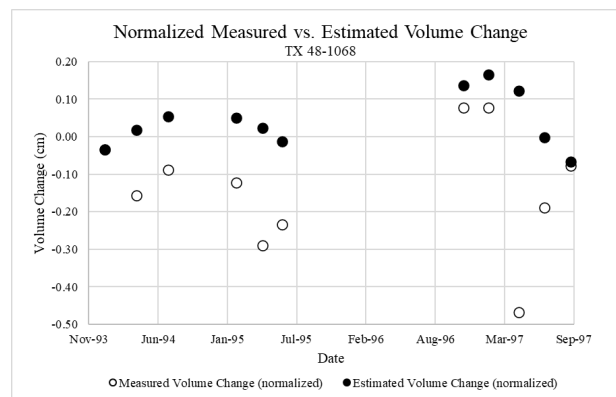


Figure 4-18: Measured vs. Estimated Volume Change Normalized to the Initial Measurement for the TX 48-1068 SMP Section

The authors are performing similar analysis with the remaining LTTP SMP sections so that calibration factors for the proposed framework can be obtained and further improve the estimation of the volume change.

4.5. 2D SSVc Estimates for Pavements

To account for covered areas, pavement or moisture barriers, a regression model was developed by Jayatilaka (1999) to estimate the vc movement using two programs developed by Gay (1994), MOPREC and FLODEF. The idea is to estimate the relationship between 1-D and 2-D vertical movement. Such analysis can be used to adjust suction profiles to consider boundary conditions, such as comparing the suction change at the edge of a covered area and the middle of the covered area, Figure 4-19, or using moisture barriers, Figure 4-20. Calibrated using data collected from ten sites located in Texas within three different climatic regions, the model is as follows:

$$\frac{VM_{2D}}{VM_{1D}} = \xi_1 \exp\left(\left(\xi_2 \frac{d}{D}\right)^{\xi_3}\right) \quad (4-39)$$

Where, VM_{2D} is the two-dimensional vertical movement from the FLODEF program, VM_{1D} is the one-dimensional vertical movement from the MOPREC program, d is the distance from the center of the pavement to the point where the vertical movement needs to be calculated in m, D is half width of the pavement in m, and ξ_1, ξ_2, ξ_3 are regression coefficients.

For pavement width less than 18.0 m:

$$\begin{aligned} \xi_1 = & 0.0561 + 1.5872(d_{am}) + 0.1244(S_m) - 0.1936 \ln(D) \\ & - 0.0007139(VM_{1D}S_m) - 0.1443(D_b d_{am}) \end{aligned} \quad (4-40)$$

$$\begin{aligned} \xi_2 = & -0.068 + 0.09134(S_m) - 0.101(D_b) - 0.000188(TMI^2) + 0.321 \ln(D) \\ & + 0.000153(VM_{1D}S_m) + 0.000706(VM_{1D}D_b) \end{aligned} \quad (4-41)$$

$$\xi_3 = \exp \left[\begin{aligned} & 1.8061 - 0.4397(S_m) + 0.4711 \ln(D) + 0.08855(D_b^2) \\ & - 0.000143(VM_{1D}TMI) + 0.003022(VM_{1D}D_b) - 1.2592(D_b d_{am}) \end{aligned} \right] \quad (4-42)$$

For pavement width greater than 22.0 m:

$$\begin{aligned}\xi_1 = & 0.376 + 0.4141(d_{am}) + 0.04078(S_m) - 0.0924(D_b) \\ & - 0.00426(VM_{1D}S_m) - 0.02584(D_b d_{am})\end{aligned}\quad (4-43)$$

$$\begin{aligned}\xi_2 = & -0.068 + 0.09134(S_m) - 0.101(D_b) - 0.000188(TMI^2) + 0.321 \ln(D) \\ & + 0.000153(VM_{1D}S_m) + 0.000706(VM_{1D}D_b)\end{aligned}\quad (4-44)$$

$$\xi_3 = \exp \left[\begin{aligned} & 3.5562 - 0.8125(S_m) + 0.3707 \ln(D) + 0.05649(D_b^2) \\ & - 0.000306(VM_{1D}TMI) - 1.6175(VM_{1D}D_b) - 0.4207(D_b d_{am}) \end{aligned} \right] \quad (4-45)$$

Where, VM_{1D} is the vertical movement from 1-D program in mm, D_b is the depth of barrier in m, d_{am} is the depth of available moisture in m, D is half width of the pavement in m, S_m is the mean suction at site in pF, and TMI is the Thornthwaite Moisture Index.

For pavement widths between 18.0 m and 22.0 m:

$$\xi = \frac{[\xi_{18}(22-D) + \xi_{22}(D-18)]}{4} \quad (4-46)$$

Where, ξ is the parameter ξ_1 , ξ_2 or ξ_3 for a pavement width of D , ξ_{18} is the parameter ξ_1 , ξ_2 or ξ_3 estimated from the equations for the pavement widths less than 18.0 m, and ξ_{22} is the parameter ξ_1 , ξ_2 or ξ_3 estimated from the equations for the pavement widths less than 22.0 m.

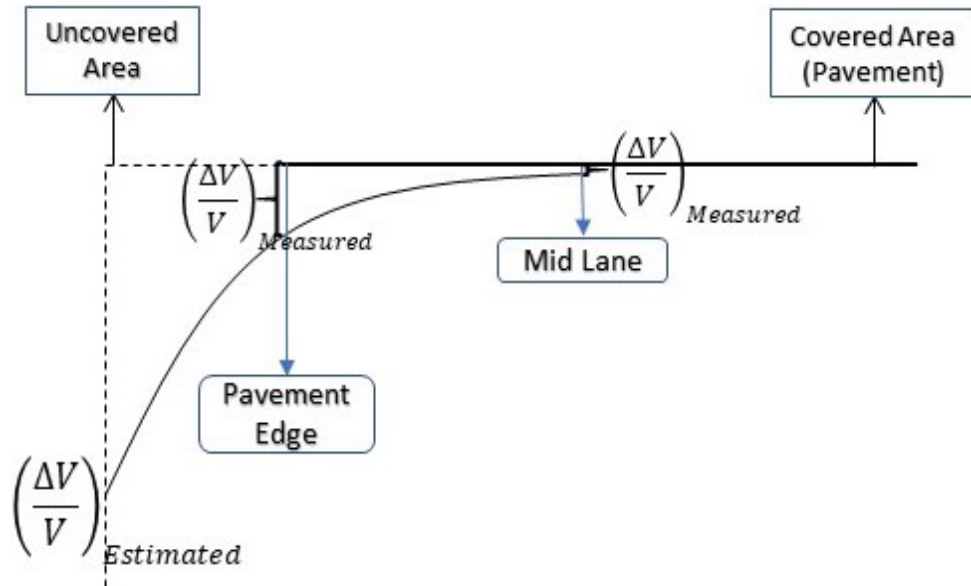


Figure 4-19 Volume Change Due to Change in Suction in Uncovered Area Compared to Different Locations under Covered Area

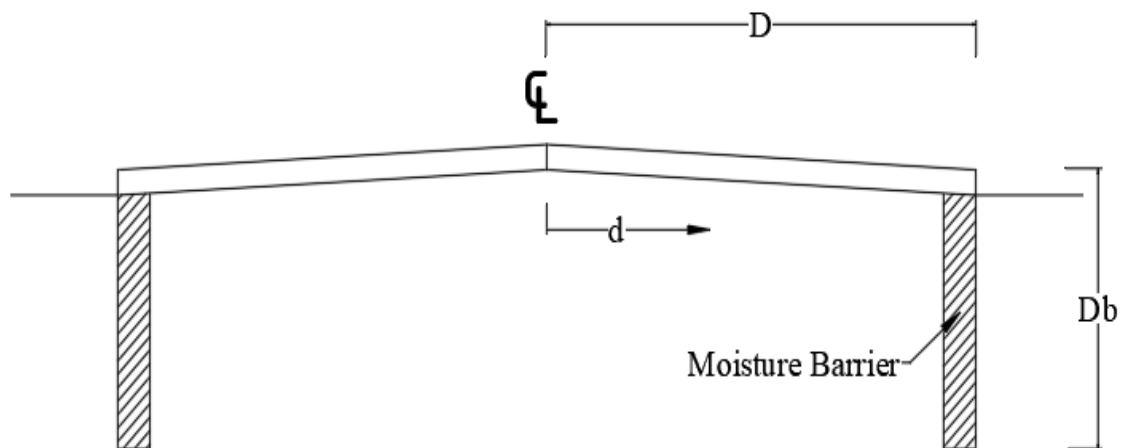


Figure 4-20 Typical Cross-Section of a Pavement with Vertical Moisture Barriers (Jayatilaka, 1999)

Lytton describes the depth of available moisture as the “maximum depth of the moisture that may be stored within the rooting depth of the soil profile.” (Jayatilaka, 1992). This definition was expanded upon by Gay in 1994:

“The depth of available moisture is the maximum depth of moisture available for use by transpiring vegetation, which is assumed to be stored within the rooting depth of the soil profile. This depth is dependent on the type and texture of the soil and the rooting depth; it is not dependent on the type of vegetation. It therefore represents the maximum depth of moisture that is lost from a soil during a transition from its wet state at field capacity to its dry state at the root potential (pF 4.2 - 4.5) of resident vegetation. Typical values for this depth have been reported to be between 5 cm and 20 cm (Thornthwaite 1948, Penman 1963); however, values as high as 27 cm have been reported for heavy clays at sites in Texas (Richardson and Ritchie 1973) and the Netherlands (Bouma and de Laat 1981).”

Gay (1994) also states that the depth of available moisture can be determined numerically by integrating a soil water characteristic curve (SWCC), over the root depth zone, between the saturation moisture content and the residual moisture content. Jayatilaka (1999) expresses the simplified triangular integration used to determine the depth of available moisture.

Gay (1994) and Jayatilaka (1999) state that if the soil-specific residual moisture content is unknown, the wilting point of vegetation (4.2 pF) or the air-dried state (5.7 pF) can be used depending on the general field conditions.

4.5.1. Effect of Moisture Barriers on SSVC Estimations

The depth of available moisture is a parameter used to determine the effect of vertical moisture barriers used in pavement design. In order to determine the sensitivity of the depth of available moisture parameter with regards to vertical moisture barriers, the empirical relationship between 1D volume change and 2D (lateral) volume change must first be understood.

The prediction of the vertical movement on expansive soils with the inclusion of vertical moisture barriers as a special provision will be based on previous studies reported in the literature. Jayatilaka (1999) suggested a regression model to estimate the relationship between one-dimensional and two-dimensional vertical movement including the effect of vertical moisture barrier depth, using the two programs MOPREC and FLODEF developed by Gay (1994). The one-

dimensional model was based on the soil deformation model proposed by Lytton (1977). The regression model was calibrated using data collected from ten sites located in Texas within three different climatic regions.

A brief description of the required input parameters used in this model are shown in Table 4-3. All the needed parameters are available either as part of the AASHTOWare ME Pavement design procedure (i.e., pavement geometry) or from the one-dimensional model results from this project. Therefore, no additional parameters will be needed to implement the model.

Table 4-3 Input Parameters Used in the Vertical Moisture Barrier Model

Input Parameter	Description
VM_{ID} (mm)	Maximum vertical movement (sum of swell and shrink)
D_b (m)	Depth of vertical moisture barrier
d_{am} (m)	Depth of available moisture
S_m (pF)	Mean suction
TMI	Thornthwaite moisture index
2D (m)	Pavement width
d (m)	Horizontal distance from the center of pavement to point of interest

4.5.2. Sensitivity Analysis of 2D SSVC Estimation

A sensitivity analysis was conducted to recognize the most critical parameters of the 2D moisture barrier model. Fixed input values of 100 mm of maximum vertical movement, 0.3 m of depth of available moisture, 3.5 pF of mean suction and 12 m of pavement width were used. Input varied in the analysis included TMI from -40 to 40; the depth of the vertical moisture barrier from 0 to 4 meters; and the mean suction value from 0 to 4 pF. These variables were compared and plotted against a normalized vertical movement. Note that the pavement edge is located at a normalized distance of one ($d/D = 1$). The results are presented in Figure 4-21 through Figure 4-23 below, show high sensitivity of the model to the three variables analyzed: TMI, moisture barrier depth and to the mean suction value.

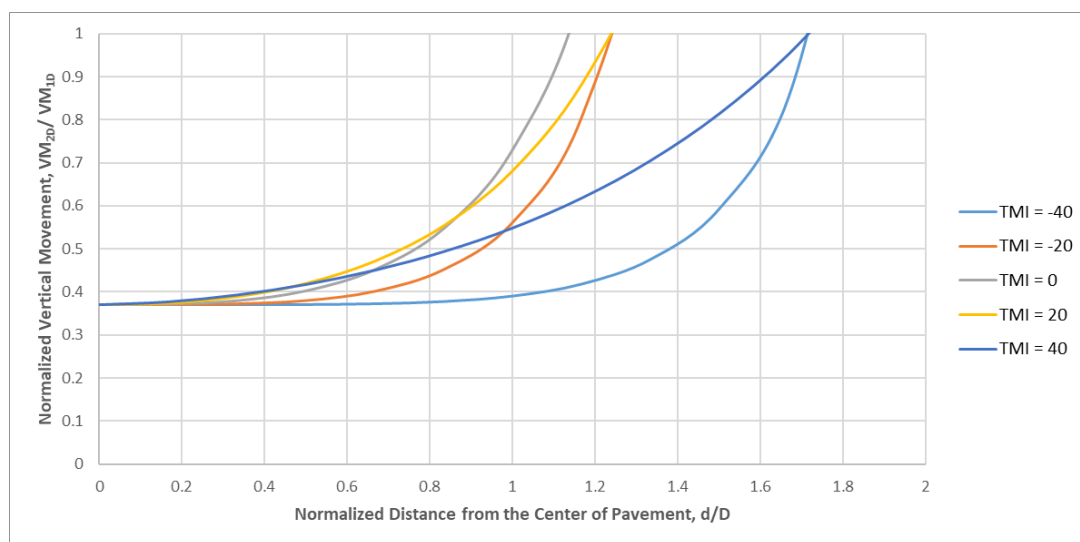


Figure 4-21 Normalized Vertical Movement as a Function of TMI

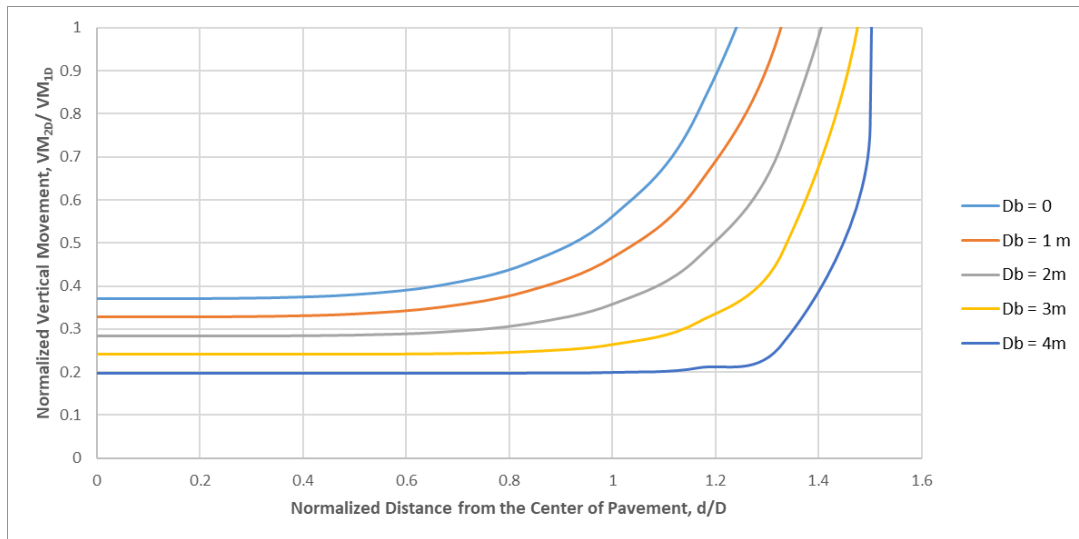


Figure 4-22 Normalized Vertical Movement as a Function of Depth of Vertical Moisture Barrier

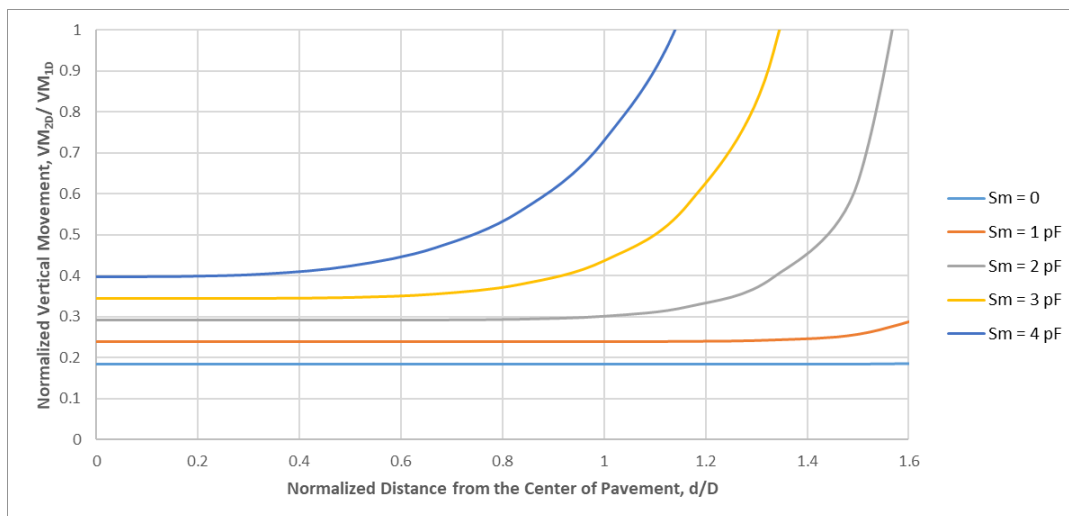


Figure 4-23 Normalized Vertical Movement as a Function of Mean Suction

The normalized vertical movement at the edge of the pavement is shown in Figure 4-24 and Figure 4-25 for different TMIs and Moisture barrier depths, respectively.

The research team is confident that this model will be extremely useful to 1) assess the vertical movement prediction away from the edge and towards the center of the pavement with and without moisture barriers; 2) as a surrogate path to calibration of the 1-D swelling predictions; and 3) to optimize design parameters.

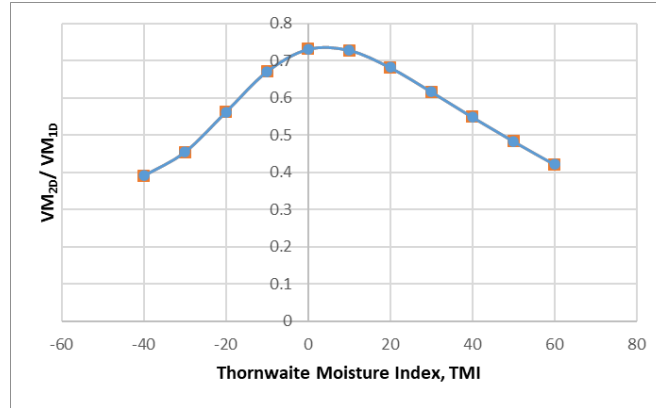


Figure 4-24 Normalized Vertical Movement at Different Thornthwaite Moisture Index at the Edge of the Pavement

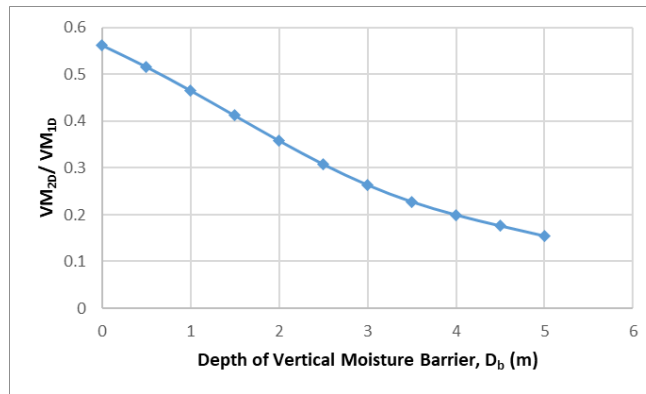


Figure 4-25 Normalized Vertical Movement for Different Depths of Vertical Moisture Barriers at the Edge of the Pavement

4.5.3. Sensitivity of the Depth of Available Moisture Parameter

Note that Gay (1994) and Jayatilaka (1992, 1999) observed that the depth of available moisture typically does not exceed 30 cm. Therefore, the first analysis performed varied the depth of available moisture from 5 cm to 50 cm. The following table, Table 4-4, summarizes the necessary parameters used in the model. Note that only the depth of available moisture was varied; all other parameters were kept constant.

Table 4-4 Input Parameters Used in the Vertical Moisture Barrier Model

Input Parameter	Value
VM_{1D} (mm)	50 mm
D_b (m)	0.9 m
d_{am} (m)	variable
S_m (pF)	4.45 pF
TMI	-46.5
2D (m)	18 m
d (m)	9 m (pavement edge)

The following figure, Figure 4-26, represents the sensitivity of the volume change at the edge of the pavement ($d/D = 1$) to the depth of available moisture parameter within the range of 5 to 50 cm.

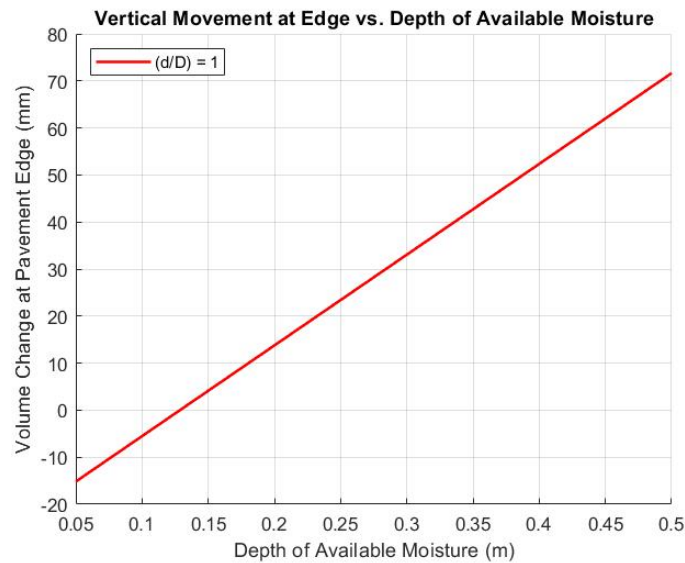


Figure 4-26 Vertical Movement at Edge vs. Depth of Available Moisture with (d/D) = 1

Note that empirical relationship results in negative volume change values (i.e. shrinkage) when the depth of available moisture is below approximately 12 cm (for this specific scenario). It is also important to note that the maximum vertical movement (VM_{ID}) of 50 mm is exceeded at the pavements edge once the depth of available moisture reaches approximately 39 cm. Based on the work on Jayatilaka (1999) and previous sensitivity analyses performed as part of this study, the lateral location of the maximum vertical movement (VM_{ID}) is typically outside the edge of the pavement and is affected by the TMI, mean suction, and depth of the vertical moisture barrier. Therefore, the volume change at the edge of the pavement is expected to be less than the maximum potential vertical movement value which would require the depth of available moisture to be less than 39 cm (for this specific scenario). This outcome agrees with observed depths of available moisture by Lytton (Jayatilaka 1992, 1999; Gay, 1994).

To ensure that the lateral location chosen for the sensitivity analysis above (pavement edge) did not affect the results, the same sensitivity analysis was performed with the location varying from the center of the pavement ($d/D = 0$) to a distance equal to half the width of the pavement away from the edge ($d/D = 2$).

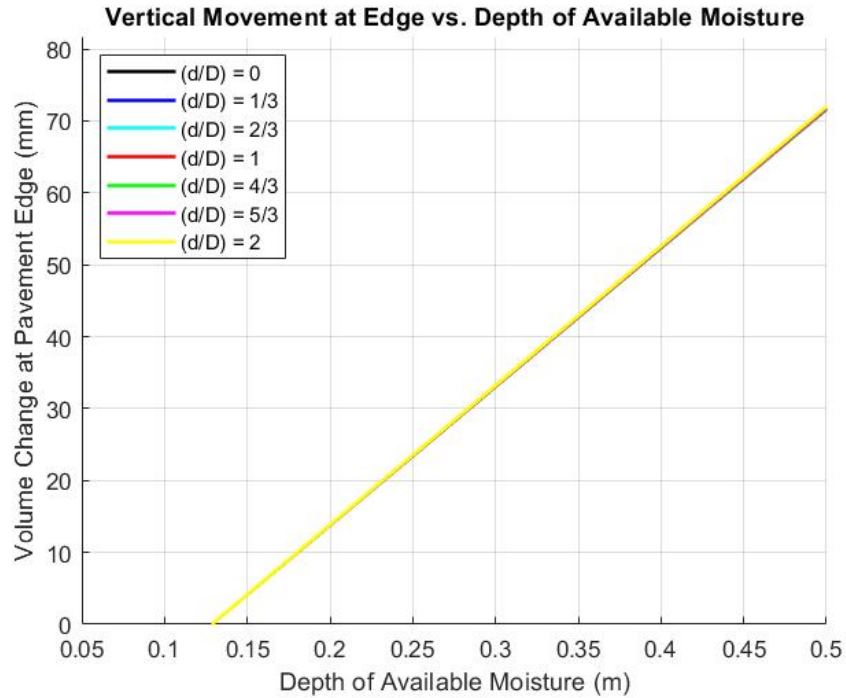


Figure 4-27 Vertical Movement at Edge vs. Depth of Available Moisture with Varying Ratios of (d/D)

It is clear from Figure 4-27 that the location chosen had no effect on the sensitivity analyses of the depth of available moisture. The effect of the depth of available moisture on the estimation of the volume change is insensitive to the lateral location within pavement profile.

The concept of the depth of available moisture can be confused with the depth of the active zone and the depth to equilibrium/stable suction. It is important to note that the depth of available moisture, as defined by Gay (1994) and Jayatilaka (1992, 1999), is not equivalent to the depth of stable suction, as defined by Vann (2019). To determine if the result of using the depth to stable suction (Vann, 2019) in place of depth of available moisture in the empirical estimation of the lateral heave reduction due to the presence of vertical moisture barriers, the previous analysis was performed a second time with an increased range of the depth of available moisture from 0.05 m (5 cm) to 4.5 m. The 4.5 m value represents an upper limit of the depth to stable suction observed by Cuzme (2018) and Vann (2019). Figure 4-28 represents the sensitivity of the Depth of Available Moisture parameter within the range of 0.05 m to 4.5 m.

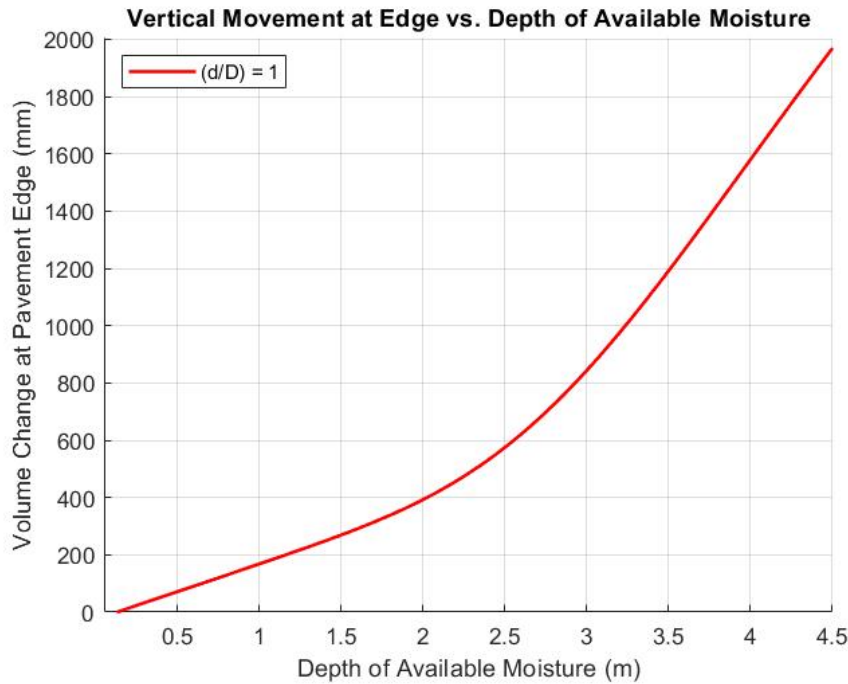


Figure 4-28 Vertical Movement at Edge vs. Depth of Available Moisture with $(d/D) = 1$

Once again, the results indicate that the magnitude of the volume change at the pavements edge is highly sensitive to the depth of available moisture parameter. If the depth to stable suction is used in-place of the depth of available moisture parameter in the analysis, the estimated volume change at the edge of the pavement will be significantly greater than the initial input of the maximum vertical movement ($VM_{ID} = 50$ mm was used in the analysis).

In summary, it has been observed that the location of the maximum vertical movement is directly affected by the depth of available moisture, provided all other parameters remain constant.

4.5.4. Back Calculation of the Depth of Available Moisture

As previously discussed, the depth of available moisture is determined from the SWCC of the soil within the root zone and knowledge of the depth of the root zone. SWCC data from all soil types is readily available and is already incorporated in the current AASHTOware software. However, data on the depth of the root zone if not readily available and may require site specific knowledge of the landscape adjacent to the roadway, which is subject to change as part of the roadway construction.

In order to eliminate the uncertainty associated with the root depth, the depth of available moisture parameter can be determined numerically if the location of the maximum vertical movement is known or estimated during the design stage. For example, a conservative assumption can be made that the maximum vertical movement will occur at the pavement edge regardless of when no vertical moisture barrier is present.

Once the location of the maximum vertical movement is assumed, an iterative calculation equivalent to the sensitivity analysis performed previously can be conducted in order to determine the depth of available moisture required for that scenario.

Table 4-5 summarizes an example scenario of a two-lane roadway with high-volume change soil, in a semi-arid climate, and variable vertical moisture barrier depth.

Table 4-5 Input Parameters Used in the Example Vertical Moisture Barrier Model.

Input Parameter	Value	Comments
VM _{1D} (mm)	140 mm	approximately 5.5 inches
D _b (m)	(0, 0.5, 1, 1.5, 2, 2.5, 3) m	Typical range or depths
d _{am} (m)	Back calculated	Based on selected location of VM_1D
S _m (pF)	4 pF	Per Vann (2019) with TMI = 10
TMI	10	semi-arid climate
2D (m)	19 m	two-lane road

Figure 4-29 presents the results of the volume change at the pavement edge based on the depth of available moisture with no vertical moisture barrier.

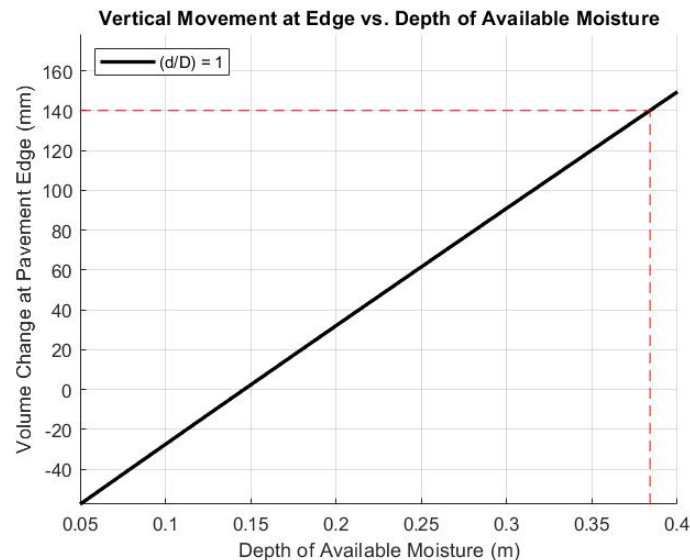


Figure 4-29 Vertical Movement at Edge vs. Depth of Available Moisture with $d/D = 1$

The red dashed lines on Figure 4-29 indicate the expected maximum vertical movement at the pavement edge when there is no moisture barrier (VM_{1D}) and the corresponding depth of available moisture.

The reduction in soil volume change due only to the pavement cover (no moisture barrier) can now be modeled using the moisture barrier factor (MBF) approach. Figure 4-30 represents the soil volume change due to the pavement covering alone.

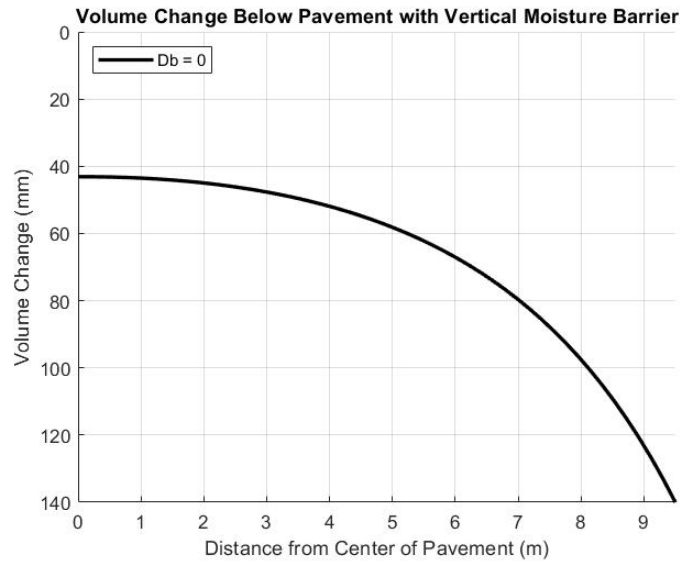


Figure 4-30 Volume Change Below Pavement with Vertical Moisture Barrier with $D_b=0$

Note that volume changed is still estimated to occur at the center of the pavement for the given scenario.

The results above are commonly normalized by the maximum vertical movement and the pavement width as shown in Figure 4-31.

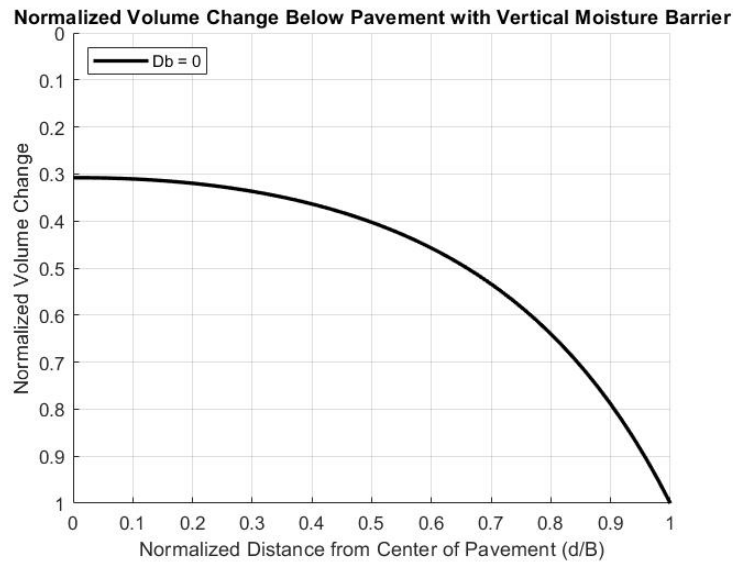


Figure 4-31 Normalized Volume Change Below Pavement with Vertical Moisture Barrier

By presenting the results in this manner, the percentage of the volume change reduction due to the pavement covering can be obtained from Figure 4-31. For this example, the covering from the pavement, without the presence of a vertical moisture barrier, results in approximately 30% of the maximum vertical movement occur at the edge of the pavement.

The effect of the depth of the vertical moisture barrier can now be determined using the back-calculated depth of available moisture and the parameters summarized in the table above. Figure 4-32 and Figure 4-33 present the raw and normalized results of the analysis.

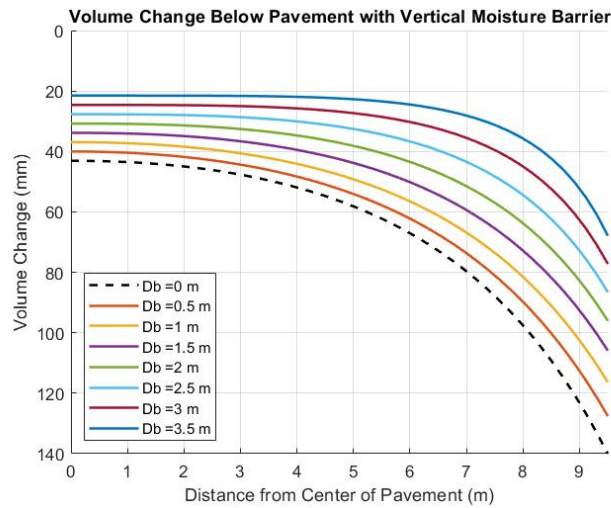


Figure 4-32 Volume Change Below Pavement with Vertical Moisture Barrier

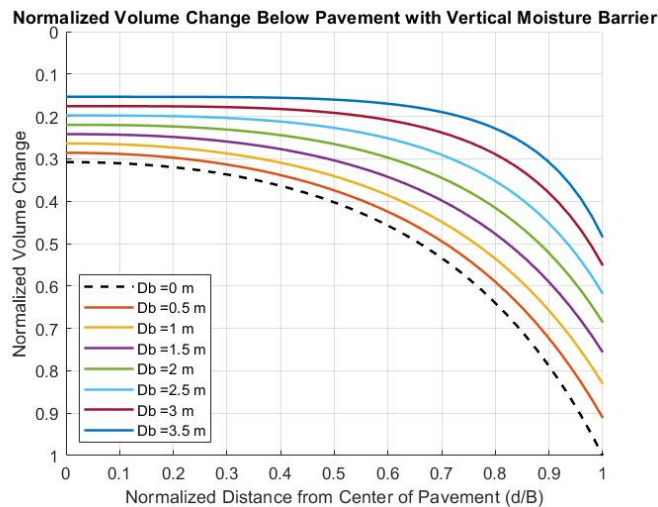


Figure 4-33 Normalized Volume Change Below Pavement with Vertical Moisture Barrier

Note that the estimated depth of stable suction (seasonal moisture fluctuation), for a site with a long term TMI = 10, is approximately 1.7 m (Vann, 2019). The results above indicate that a moisture barrier depth greater than the depth of stable suction (2 m) still results in vertical movements near the center of the pavement that are approximately 20% of the expected maximum vertical movement at the edge of the pavement when no barrier exists.

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